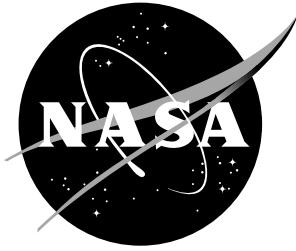


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System Risk Assessment and Allocation in Conceptual Design

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May 2003

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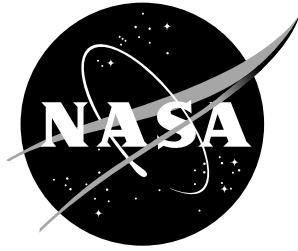
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ABSTRACT

As aerospace systems continue to evolve in addressing newer challenges in air and space transportation, there exists a heightened priority for significant improvement in system performance, cost effectiveness, reliability, and safety. In order to address system complexity, aerospace design is decomposed along disciplinary specialty lines. To complete system design, tools must be available for coordination or re-integration of the individual disciplinary efforts. Within this process, a probabilistic methodology is needed to include various types of uncertainties and to assess and design for system reliability. Finally, to achieve design improvements, optimization tools are also necessary. Tools, which synthesize multidisciplinary integration, probabilistic analysis, and optimization, are therefore needed to facilitate design decisions allowing trade-offs between cost and reliability.

This study investigates tools for probabilistic analysis and probabilistic optimization in the multidisciplinary design of aerospace systems. A probabilistic optimization methodology is demonstrated for the low-fidelity design of a reusable launch vehicle at two levels, a global geometry design and a local tank design. Probabilistic analysis is performed on a high fidelity analysis of a Navy missile system. Furthermore, decoupling strategies are introduced to reduce the computational effort required for multidisciplinary systems with feedback coupling.

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CHAPTER I

INTRODUCTION

The maturity of the aerospace industry is reflected in the modern challenge of engineering increasingly complex systems. At the same time, there exists a heightening priority to assess and design for cost effectiveness and reliability. As a result, design goals for future generation systems are being expressed in terms of risk and cost. For example, goals for next generation reusable launch vehicles (RLV) include improving safety to achieve a less than 1/10,000 risk of crew loss and reducing cost to \$1,000/lb or less for payload delivery (NASA 2000). These goals represent a ten-fold improvement in cost and a one hundred-fold improvement in the reliability over the current space shuttle system. Whether they are achievable remains to be seen; first, multiple design approaches and several emerging technologies need to be evaluated and compared.

Assessing risk and cost for new systems with emerging technologies is extremely difficult. Historical databases based on legacy systems have questionable application to novel systems. Analytical models are an alternative but have their own limitations. A traditional approach to design (in which disciplinary analysis is done independently and in sequence) would be extremely expensive and time consuming for this kind of comparative evaluation. Furthermore, risk design requirements given in probabilistic terms (i.e. required probabilities of success or failure) cannot be targeted through conventional deterministic design methods (which deal with risk in terms of factors of safety). Thus a design process that effectively balances cost and risk needs to

incorporate multidisciplinary, concurrent design techniques along with probabilistic analysis in an optimization framework.

To meet modern aerospace design goals (such as those for the 2nd generation RLV), a multidisciplinary integration and optimization strategy is needed. In the early conceptual stage, designs need to be evaluated more quickly when less detail is available. At this stage, lower fidelity models may be acceptable (assuming these models do not sacrifice essential performance characteristics) in order to identify the most attractive designs for further development. As designs are developed in more detail (i.e. during preliminary and detailed design phases), higher fidelity analysis tools can be employed. For example, response surface models for system performance are used during the early stages of conceptual design; they allow for quick analysis and easy optimization. Integration of higher fidelity analysis codes across disciplines is more challenging. These codes require a greater computational effort, a problem that is exacerbated by optimization and probabilistic analysis algorithms that require multiple iterations. Also, the inter-disciplinary coupling of higher fidelity analyses creates further complications. At the later stages, more expensive analysis through wind tunnel experiments and component level tests are appropriate; this information needs to be synthesized from a multidisciplinary system perspective as well. Finally, the sensitivity of design elements to overall cost and risk influence the level of fidelity required at each stage of development.

Decisions that have the greatest effect on system risk and cost are made during the conceptual design stage. For this reason, despite the lower fidelity of the tools used, cost and risk evaluation and optimization needs to be a vital part of conceptual design

analysis. It is also recognized that such design decisions need to account for the uncertainties in design variables that significantly impact cost and risk. However, current methods of system performance and risk assessment, capable of including uncertainty effects, are typically employed only after the system has been designed in detail, and in some cases, manufactured and tested. This stage is quite late in the system development process when it is difficult to make adjustments that would significantly reduce either cost or risk. Therefore, reliability analysis methods that can be applied during conceptual design and incorporated in a multidisciplinary system are needed.

In order to effectively incorporate probabilistic analysis in a progressive design strategy that builds from conceptual design (with low fidelity models) to detailed design (with high fidelity models), the following capabilities are needed during the early stages of design:

- (1) Failure sequence diagram construction (Karamchandani, 1987; Swaminathan and Smidts, 1999; Thoft-Christensen and Murotsu, 1986) without detailed information, based on initial concepts and approximate component descriptions;
- (2) Probabilistic system reliability analysis (Haldar and Mahadevan, 2000; Mahadevan and Dey, 1997; Mahadevan and Liu, 2002)
- (3) Sensitivity analysis linkages between system events, component events, and major design variables (Ryan, 1993; Mahadevan et al, 1997)
- (4) Risk vs. cost trade-off computations at component level and system level (Cruse et al, 1994; Frangopol, 1985)

- (5) Risk allocation using a combination of optimization and Bayesian statistical analysis methods (Martz and Waller, 1982; Mahadevan et al, 2001; Mahadevan and Liu, 1998; Mahadevan, 1992).

These capabilities are being developed for the 2nd generation RLV conceptual design and a Navy hypersonic missile design at Vanderbilt University in collaboration with researchers at NASA Langley Research Center. In this report, capabilities two through four are demonstrated for three related applications: a conceptual multidisciplinary RLV design based on response surfaces, a structural component design for an RLV liquid hydrogen tank, and a higher fidelity multidisciplinary analysis of a Navy missile system.

Multidisciplinary Optimization in Aerospace Design

Aerospace design is a complex process requiring analysis and optimization across multiple disciplines. In many cases, relatively mature (high and low fidelity) disciplinary analysis tools are available. These disciplinary analyses cannot be taken in isolation since they are coupled to one another through shared input and output. Furthermore, system design objectives and constraints may span several disciplines. Thus, a design process needs to address issues of the coupled multidisciplinary system. Integrating disciplinary analyses into a multidisciplinary framework and finding practical ways to solve system optimization problems is a serious challenge.

The first aspect of the multidisciplinary design challenge lies in building a system analysis from a collection of disciplinary analyses. The system architecture must be thoroughly understood in order to identify how the individual disciplines are coupled. In

addition to this technical challenge, organizational and/or computational difficulties often present the largest obstacles in creating a multidisciplinary analysis. The second aspect of the challenge is in solving system optimization problems for the purpose of design. Here, the complexity, stability, and efficiency of the system analysis have significant consequences.

Several studies have addressed RLV design as a multidisciplinary optimization problem (MDO) (Unal et al, 1998a; Braun et al, 1995; Braun and Moore, 1996; Unal et al, 1998b), and developed strategies on a number of fronts. One front may be considered the ‘design’ environment. This includes both organizational (Monell and Pinland, 2000) and computational (Salas and Townsend, 1998; Padula et al, 1999; NASA et al, 2001) frameworks that allow automation of multidisciplinary analyses. For example, the integration software, Model Center® (Phoenix Integration, 2002) provides the framework for the multidisciplinary analysis of the missile system presented in Chapter 4. A second front for multidisciplinary design research is the multidisciplinary optimization (MDO) problem formulation. A review of MDO problem formulations and implementation issues is given in Alexandrov and Kodiyalam (Alexandrov and Kodiyalam, 1998).

In addition to the design environment and the MDO formulation, a third front for MDO research includes approximation modeling. These models are useful in allowing trade-offs between analysis fidelity and computational complexity at the early stages of design. This concept can also be applied to a single multidisciplinary optimization. For example, in approximation/model management optimization (AMMO) (Alexandrov and Lewis, 2000), a trust region approach is used to approximate a system objective with a low-fidelity model, update the model at each optimization iteration, and resort to higher

fidelity approximations when needed. Various methods are available to build these initial, low-fidelity models including response surface models, Taylor series approximations, splines, and Kriging models. In particular, a D-optimal response surface strategy has been implemented for a RLV sizing application using 3 disciplines: geometry, weights, and aerodynamics (Unal et al, 1998a). This response surface formulation provides the baseline for the probabilistic optimization approach presented in the second chapter.

A final area of concern for aerospace design research is the optimization objective. As stated earlier, program goals have been expressed in terms of cost objectives (e.g, \$1,000/lb payload) and risk objectives (e.g, 1/100 risk of crew loss). It is logical, then, that the design optimization should incorporate these goals. These goals could be incorporated through either the optimization objective function or the constraints. In both the RLV and missile designs, minimizing other parameters considered roughly proportional to cost, often approximates cost optimality. For example, Braun and Moore (1996) compare a collaborative optimization analysis using four different optimization objectives: development cost, gross lift-off weight, vehicle empty weight, and ΔV (velocity change from the rocket equation).

Incorporating risk goals explicitly in the optimization is a concept that needs significant development in multidisciplinary aerospace design. Risk is a concept that is difficult to measure and challenging to model. A traditional approach is to model risk via factors of safety. Factors of safety provide a qualitative measure of risk, but cannot be translated to probabilistic metrics (such as 1/100 risk of crew loss). Another tactic is to model risk from empirical data obtained through testing or historical records.

For example, NASA has developed a reliability and maintainability analysis and estimation tool (RMAT), which models RLV operations and supports reliability through a comparison to historical database records from current operational aircraft (Unal et al, 1998b). Response surfaces are generated from RMAT for use in the optimization of performance reliability. Unfortunately, historical records have limited use for novel designs, and testing is prohibitively expensive for complex systems.

An alternate approach is to estimate risk through simulation of physics-based models by explicitly considering the uncertainty in the input parameters or in the model itself. Deterministic approaches that have been the focus of design optimization methods thus far fail to account for these uncertainties. Therefore a probabilistic approach is needed for risk-based optimization under uncertainty and is pursued in this study.

Probabilistic Analysis and Optimization

Aerospace design optimization to date has been based on a deterministic approach. Input variables are assumed to be non-varying and the system is assumed to behave exactly as an analysis model predicts. In this case, the analysis output will be deterministic, and there will not be any uncertainty-based metric for assessing risk. A probabilistic approach, on the other hand, allows for random variation in the input variables and can also consider the error between model predictions and true system behavior. In this case, the output will also have random variation. This random variation is characterized by a probability distribution function (PDF) defined by its shape and a set of distribution parameters (e.g. mean and standard deviation). Risk objectives can then be defined in terms of this output uncertainty (Haldar and Mahadevan, 2000). A

probabilistic optimization will then characterize uncertain objectives and constraints in terms of probability distribution parameters and statistics (e.g. minimize μ_z subject to $P(x_1 < x < x_2) \leq p_f$, where μ_z is the mean of output z and p_f is some acceptable probability of failure for x to be in the interval $[x_1, x_2]$).

In the physics-based limit-state method for reliability modeling, a performance function (g_i) is defined in terms of the random variables corresponding to a failure criterion. Typically, $g_i < 0$ represents failure, $g_i > 0$ represents success, and $g_i = 0$ is referred to as the limit state (Haldar and Mahadevan, 2000). The probabilities of failure may be evaluated with a number of probabilistic techniques including simulation methods (e.g. Monte Carlo, adaptive importance sampling, Latin Hypercube sampling, etc.) or analytical approximation methods (First Order Reliability Method (FORM), Second Order Reliability Method, First Order Second Moment Methods) (Haldar and Mahadevan, 2000). For optimization problems, an analytical approximation method is preferred due to computational efficiency.

Probabilistic optimization has been the focus of several recent research efforts. Optimization formulations have three components (design variables, the objective function, and constraints) leaving a few alternatives for stating the probabilistic problem. First, input variables (including design variables) are uncertain but can be characterized by their distribution parameters. Thus optimization problems may have design variables that include mean values, standard deviations, a combination of both, or other parameters describing their randomness (Kowal and Mahadevan, 1998). (In addition to the design variables, other input variables may also be modeled as random variables. However, this does not complicate the optimization process, since all random variables are considered

together in the evaluation of the probabilistic constraint.) Second, the objective function may be in terms of failure probability (e.g. minimize probability of failure), an independent cost or performance measure (e.g. minimize mean development cost), or may be a cost or performance measure that depends on reliability (e.g. minimize Total Cost = Development Cost + Failure Cost * Probability of Failure). Finally, constraints may include either cost or reliability design goals that are not considered in the objective function.

Another consideration for the probabilistic optimization formulation addresses the level of the failure. Physics-based limit states usually represent component-level events, where probabilities of failure can be calculated fairly easily. System-level failure results from component failures in series, parallel, or a combination of both. Event sequence diagrams may be used to define the system. For large systems with non-independent components, calculation of system failure probabilities can be time consuming. The branch and bound method has been used to streamline the system calculation (Thoft-Christensen and Murotsu, 1986), but it is still difficult to employ within an optimization procedure. Optimum design of framed structures has been undertaken, using component-level reliability constraints and a system-level objective function (weight of system structure) (Frangopol, 1985; Mahadevan, 1992). Oakley et al (1998) combine stochastic optimization with adaptive response surfaces. This work considers two types of uncertainty (variability of operating conditions and uncertainty in extreme values) under a system of multiple load cases. Leheta and Mansour (1997) apply a system reliability optimization formulation to structural design of marine stiffened panels. They employ the First Order Reliability Method to determine component reliability and use an

approximation for the reliability of a series system that includes multiple failure mechanisms.

Probabilistic optimization problems (also called reliability-based design optimization or RBDO) have been solved through a number of techniques. These techniques employ standard linear or non-linear programming algorithms and analytical reliability analysis methods to solve carefully formulated probabilistic problems. In the reliability index approach (RIA), probabilistic constraints are reformulated in terms of reliability indices. In the performance measure approach (PMA), the constraints are transformed through an inverse reliability analysis. RIA and PMA methods are combined in an adaptive scheme that uses RIA when the probabilistic constraint is active (i.e. the design point is on the constraint boundary) and PMA when the constraint is inactive (Oakley et al, 1998). In Putko et al (2001), a methodology using sensitivity derivatives is presented for design optimization using computational fluid dynamics (CFD) codes. Tryon et al (2002) also present a sensitivity-based optimization framework, which they demonstrate on a vibration control problem. In Royset et al (2001), a decoupling approach is presented for three types of probabilistic optimization problems; in this methodology, constraints originally given in terms of reliability indices are replaced by a function representing the limit state minimum within a hypersphere of specified radius.

In the applications presented in this report, probabilistic formulations are chosen which use mean values as the design variables and probabilities of failure as constraints. A direct (coupled) approach using the first order reliability method (FORM) is chosen for the reliability analysis.

Reliability-Based Multidisciplinary Optimization

The goal of this study is to present a strategy for conceptual system design that simultaneously addresses both cost and risk requirements. The foundation of this strategy is reliability-based multidisciplinary optimization (or multidisciplinary optimization under uncertainty), a technique that must combine MDO methods and probabilistic analysis. In this case, computational effort is a major concern since the direct approach involves nested loops for multidisciplinary analysis, probabilistic analysis, and optimization. Three applications are given for demonstration. The first application, presented in the following chapter, is the global design of a reusable launch vehicle based on geometry, weights, and aerodynamic disciplines. In this example, a low-fidelity approach is taken by reducing the multidisciplinary analysis through response surfaces. In Chapter III, the global design result is carried forward in local design, i.e. structural sizing of a liquid hydrogen tank. This application demonstrates the integration of local and global designs; it also uses system reliability concepts to represent multiple failure modes. Next, Chapter IV highlights the unique challenges for probabilistic optimization created by coupled multidisciplinary systems, and presents techniques for reducing the computational effort for these systems. In Chapter V, probabilistic analysis is performed on a five-discipline analysis of a hypersonic missile system. Here, the difficulties of using higher fidelity multidisciplinary analyses are made evident and directions for future research are suggested. The final chapter summarizes the work presented and recommends directions for future investigation.

CHAPTER II

PROBABILISTIC GLOBAL DESIGN OF A REUSABLE LAUNCH VEHICLE

In this chapter, the probabilistic optimization methodology is demonstrated for the global design of a reusable launch vehicle. This is done by solving an optimization problem, which is formulated with a cost-related objective (vehicle empty weight) and risk-based constraint (probability of satisfying a pitching moment condition). The system analysis is based on three disciplines: geometry, weights, and aerodynamics. Response surfaces are used to consolidate the multidisciplinary analysis to facilitate the probabilistic optimization. The analysis and optimization take into account the uncertainties in design variables that have important effects on safety, performance, and cost.

This work builds upon a deterministic optimization application for RLV conceptual design originally developed by Unal et al (1998a); the deterministic problem is re-formulated for probabilistic system reliability analysis and optimization. In the first section of this chapter, the deterministic problem is explained in detail. This is followed by a section describing the probabilistic re-formulation including details of modeling uncertainty. In the “Results and Discussion” section, optimal designs, based on several constraint criteria, are then compared with those of the deterministic problem. This section also includes an analysis of probabilistic sensitivities. Finally, some specific conclusions are made regarding this application of probabilistic design.

Launch Vehicle Conceptual Design: Deterministic approach

As a first effort in the RLV multi-disciplinary design analysis, low fidelity second-order response surface models have been developed for a sizing analysis of a wing-body, single stage-to-orbit vehicle (Unal et al, 1998). For this application, a launch vehicle is sized to deliver 25,000 lb in payload from the Kennedy Space Center to the International Space Station. The selected vehicle geometry (shown in Fig. 1) has a slender, round fuselage and a clipped delta wing. Elevons provide aerodynamic and pitch control. Vertical tip fins provide directional control and body flaps provide additional pitch control.

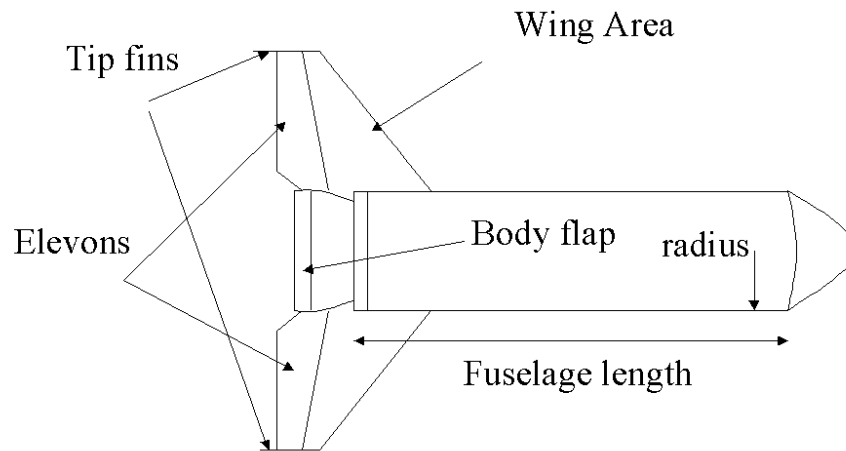


Figure 1: RLV Conceptual Design

As a first step in the conceptual design, three disciplines (geometry, weights/sizing, and aerodynamics) are considered in a constrained optimization problem. A vehicle geometry that minimizes mean dry weight is expected to minimize overall cost, so this is chosen as the objective function. For stability, the pitching moment (C_m) for the vehicle should be zero or extremely close to zero. In addition, C_m should decrease as the angle of attack increases. This is achieved by adjusting the control surfaces to trim the

vehicle as the angle of attack is increased. Thus the aerodynamic analysis for pitching moment constrains the optimization.

The optimal vehicle design is determined by six design variables: fineness ratio (fuselage length / radius), wing area ratio (wing area / radius²), tip fin area ratio (tip fin area / radius²), body flap area ratio (body flap area / radius²), ballast weight fraction (ballast weight/ vehicle weight), and mass ratio (gross lift-off weight/ burnout weight). The design variables are summarized in Table 1. For the aerodynamic part of the analysis, three additional variables are required to describe the adjustment of control surfaces in order to trim the vehicle: angle of attack, elevon deflection, and body flap deflection. The pitching moment constraint must hold during all flight conditions; nine flight scenarios (constructed with three velocity levels and three angles of attack) are used as a representative sample. The representative velocities (Mach 0.3, Mach 2, and Mach 10) were selected as those originally used in Unal, et al (1998a) for which response surfaces have already been generated. The control variables used for each of the nine scenarios are given in Table 2. The deterministic optimization problem may then be written as follows: *‘Minimize vehicle dry weight (W) such that the pitching moment coefficient (C_m) for each of 9 scenarios is within acceptable bounds [- 0.01,+0.01].’*

Table 1: Design Variables

Design variables	Lower bound	Upper bound
Fuselage finess ratio (fr)	4	7
Wing area ratio (war)	10	20
Tip fin area ratio (tfar)	0.5	3
Body flap area ratio (bfar)	0	0
Ballast wt fraction (bl)	0	0.04
Mass Ratio (mr)	7.75	8.25

Table 2: Control Variables

	Nominal Angle of Attack		Minimum Angle of Attack		Maximum Angle of Attack	
	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Aero control variables						
Mach 0.3						
Angle of attack, deg	12	12	5	5	15	15
Elevon deflection, deg	-15	10	-15	10	-15	10
Mach 2.0						
Angle of attack, deg	10	10	5	5	15	15
Elevon deflection, deg	-15	10	-15	10	-15	10
Body flap defl, deg	-10	10	-10	10	-10	10
Mach 10						
Angle of attack, deg	30	30	25	25	40	40
Elevon deflection, deg	0	30	0	20	0	20
Body flap defl, deg	0	20	0	20	0	20

This is a multidisciplinary problem requiring the synthesis of information from three analysis codes: a geometry-scaling algorithm, a weights and sizing code (CONSIK) and an aerodynamic code (APAS). Integrating these codes is somewhat complex since, in addition to the variables already described, there are numerous, non-design variables that are passed between the codes. This effort would be computationally expensive for an optimization algorithm. Thus Unal et al (1998a) conducted two over-determined D-optimal experimental designs to reduce these analyses to a set of second order response surface models that relate the vehicle dry weight (W) to the 6 geometry variables, and the pitching moment coefficient (C_m) to both the geometry and control variables. (Note: There are 30 independent variables in total. The control variable values are specific to a scenario. All 9 scenarios involve adjusting the angle of attack and elevon deflection but

only the supersonic and hypersonic response surface models specifically include body flap deflection as one of the statistically significant random variables.)

In the design of experiments for the weight function, forty-five multidisciplinary analyses were conducted according to the design matrix described in Table 3. For each row in the matrix, an analysis is run with design variable values given according to their corresponding column of normalized values. Cells in the matrix with a value of -1 denote the lowest variable value; those with a value of zero denote the midrange value; and those with a value $+1$ denote the highest variable value. These do not necessarily correspond to the constraint bounds used in the optimization but rather depict the range of values over which the response surfaces are valid. It is important that the optimum domain be within the domain of the response surfaces. If the response surfaces lead to an optimum domain outside the range of the response surfaces, then an adaptive “trust region” strategy may be needed (Zou et al). It is also desirable to check the optimum solution with a full high-fidelity analysis if possible. Also, in the initial conceptual design stage, the feasible response surface ranges may be determined using expert opinion. In this case the optimization domain is the same as the response surface domain except that the body flap area is additionally constrained.

Curve-fitting the experiment results produces a 2nd degree polynomial for the dry weight of the vehicle. A similar methodology was used to generate 2nd degree response surfaces for pitching moment coefficient. The response surface polynomials are expressed in terms of the normalized variables from the D-optimal matrix (values between -1 and 1), which we will refer to as reduced variables. These are distinguished from the original variables given in Tables 1 and 2.

Once the response surfaces were generated, a gradient-based non-linear optimizer was used to solve the optimization problem. A verification run of the multidisciplinary analysis was performed at the optimal design point. The result agreed with the response surface prediction within 1%.

Table 3: 6 Parameter D-Optimal Design Matrix (Unal et al, 1998a)

	FR	WA	TFA	BFL	BL	MR
1	-1	-1	-1	-1	-1	1
2	-1	-1	-1	-1	1	-1
3	-1	-1	-1	1	-1	0
4	-1	-1	-1	1	1	1
5	-1	-1	0	1	-1	-1
6	-1	-1	1	-1	-1	1
.
.
.
.
41	1	1	0	0	-1	1
42	1	1	1	-1	-1	-1
43	1	1	1	-1	1	1
44	1	1	1	1	0	-1
45	1	1	1	1	1	1

Launch Vehicle Conceptual Design: Probabilistic Approach

In order to reformulate the above problem in probabilistic terms, the problem is restated as *‘Minimize mean weight such that the pitching moment coefficient for all 9 scenarios has a low probability (less than 0.1) of failing to be within the acceptable bounds [-0.01, +0.01].’* Note here that the output parameters (weight, W and pitching moment, C_m) are random variables. They cannot be known exactly since the inputs and analysis model, from which they are based, are subject to uncertainty. Therefore

minimizing the mean weight approximates the weight minimization, and the pitching moment constraint is estimated as a probability of failure to be within acceptable bounds. The solution of this revised, probabilistic problem consists of four parts: 1) characterization of random input variables, 2) defining the limit states for failure, 3) probabilistic analysis, and 4) optimization.

Characterization of Random Input Variables

Original Variables

The first step is to define the uncertainty of the input variables. From the standpoint of the conceptual analysis, these distributions will represent statistical variation in input parameters. However, it is not obvious how this information shall translate into physical phenomena. For fully designed system, these distributions would represent uncertainty between as-built dimensions and that specified by the design. However, details from the conceptual design will inevitably undergo numerous revisions prior to any kind of physical construction; any relationship between the conceptual design and the as-built condition will be superceded by intermediate higher fidelity analysis and design. Enough data is not yet available to characterize the physical uncertainty of the as-built dimensions. However, at this stage, a high level of precision is not required. Therefore, approximate data was constructed based on expert opinion (the experts were asked to estimate uncertainty of the as-built dimensions based on a full design.) All distributions were assumed normal (a simplifying assumption to facilitate analysis). However, any other distribution can be easily included within the probabilistic

framework used here. The mean values (μ) of the variables are the design variables and change during the optimization iterations. The standard deviations (σ) were assumed to be either one third the range or were estimated from an assumed coefficient of variation (δ) where $\sigma = \delta * \mu$. Table 4 describes the characterization of these original input variables.

Reduced Variables

Since these variables are being computed through linear functions of the original variables, their distribution parameters can be easily found. For example, the original variable for fuselage finess ratio, fr , has a range from 4 to 7. The response surfaces use a

reduced variable, $FR = \frac{fr - 5.5}{1.5}$, where 5.5 is the midrange value and 1.5 is the half-

range. FR will have a mean $\mu_{FR} = \frac{\mu_{fr} - 5.5}{1.5}$, and standard deviation $\sigma_{FR} = \frac{\sigma_{fr}}{1.5}$.

Table 5 describes the reduced variables.

Model Error

In a physics-based reliability model, only a portion of the uncertainty in a system is accounted for through its input parameters. The analytical model for the RLV is a regression model (the response surfaces) of a computational model (the multidisciplinary system analysis) of a conceptual model (based on principles of physics, usually in the form of partial differential equations) of the real system. Regardless of variation in input, the conceptual model does not predict real system behavior exactly, the computational model does not exactly represent the conceptual model, and the regression model

introduces further error. This regression error can be determined from the residuals of the least squares, but it is only one of many sources of error introduced by the model. In addition, the response surfaces were constructed by assuming deterministic values for many input parameters that in reality are uncertain. Furthermore, “biases” are present in the computational model due to system dynamics not captured by the model. Reinelt et al (2002) and Ninness and Goodwin review various strategies for quantifying errors and uncertainty in the errors of computational models. Oberkampf and Trucano (2002) also give a thorough review of errors associated with computational fluid dynamics models and their significance in model validation.

Experimental information is not available during the initial stages of design. Also, the fidelity requirements at this state may not warrant sophisticated error modeling. However, in some cases, tolerances on the other input variables may be so tight as to make them effectively deterministic compared to the effect of model error. Thus, it is useful to at least estimate model error in conceptual probabilistic analysis. Several methods for the probabilistic quantification of model error arising from various levels of approximation are currently under development (Rebba, 2002). In this application, the model “error” is considered simply as an additional random variable applied to the predicted pitching moment as a percentage. Here, model error has a mean of 0 and a standard deviation of 10%. Thus, the output pitching moment coefficient, $C_m = C_m(pred) * (1 + error)$, where $C_m(pred)$ is the pitching moment predicted from the response surfaces.

Table 4: Statistics of Original Variables

Design variables	Distribution	Mean	range	COV	std dev
<i>Geometry Variables</i>					
Model Error	Normal	0			0.1
Fuselage finess ratio, fr	Normal	6.2796		0.01	0.0628
Wing area ratio, wa	Normal	16.1524		0.01	0.1615
Tip fin area ratio, tfa	Normal	0.5		0.01	0.005
Body flap area ratio, bfa	Normal	0		0.01	0
Ballast wt fraction, bl	Normal	0		0.01	0
Mass Ratio, mr	Normal	7.75	0.05		0.0167
<i>Control Variables</i>					
Scenario 1: Mach 0.3 - Nominal angle of attack					
Angle of attack	Normal	12	0.1		0.0333
Elevon Deflection	Normal	-7.7336	1		0.3333
Scenario 2: Mach 0.3 - Minimum angle of attack					
Angle of attack	Normal	5	0.1		0.0333
Elevon Deflection	Normal	-1.3109	1		0.3333
Scenario 3: Mach 0.3 - Maximum angle of attack					
AOA-Mach 0.3 Max	Normal	15	0.1		0.0333
Elevon Deflection	Normal	-10	1		0.3333
Scenario 4: Mach 2 - Nominal angle of attack					
Angle of attack	Normal	10	0.25		0.0833
Elevon Deflection	Normal	-0.3066	1		0.3333
Body flap	Normal	-0.3481	1		0.3333
Scenario 5: Mach 2 - Minimum angle of attack					
Angle of attack	Normal	5	0.25		0.0833
Elevon Deflection	Normal	-5.0628	1		0.3333
Body flap	Normal	-5.0156	1		0.3333
Scenario 6: Mach 2 - Maximum angle of attack					
Angle of attack	Normal	15	0.25		0.0833
Elevon Deflection	Normal	-0.1877	1		0.3333
Body flap	Normal	10	1		0.3333
Scenario 7: Mach 10 - Nominal angle of attack					
Angle of attack	Normal	30	0.25		0.0833
Elevon Deflection	Normal	29.8082	1		0.3333
Body flap	Normal	6.4322	1		0.3333
Scenario 8: Mach 10 - Minimum angle of attack					
Angle of attack	Normal	25	0.25		0.0833
Elevon Deflection	Normal	20	1		0.3333
Body flap	Normal	20	1		0.3333
Scenario 9: Mach 10 - Maximum angle of attack					
Angle of attack	Normal	40	0.25		0.0833
Elevon Deflection	Normal	20	1		0.3333
Body flap	Normal	20	1		0.3333

Table 5: Statistics of Reduced Variables

Design variables	Mean	Std dev
<i>Geometry Variables</i>		
Model ERROR - Xe	0	0.1
FR - X1 (normalized fuselage finess ratio)	0.5198	0.0419
WA - X2 (normalized wing area ratio)	0.2305	0.0323
TFA - X3 (normalized tip fin area ratio)	-1	0.004
BFL - X4 (normalized body flap area ratio)	-1	0
BL - X5 (normalized ballast weight fraction)	-1	0
MR - X6 (normalized mass ratio)	-1	0.6667
<i>Control Variables</i>		
Scenario 1: Mach 0.3 - Nominal angle of attack		
alpha (derived from angle of attack) –X7	0.9512	0.0159
delev (derived from elevon deflection) – X8	-0.6130	0.0264
Scenario 2: Mach 0.3 - Minimum angle of attack		
alpha (derived from angle of attack) –X7	-2.378	0.0159
delev (derived from elevon deflection) – X8	-0.1039	0.0264
Scenario 3: Mach 0.3 - Maximum angle of attack		
alpha (derived from angle of attack) –X7	2.378	0.0159
delev (derived from elevon deflection) – X8	-0.7927	0.0264
Scenario 4: Mach 2 - Nominal angle of attack		
alpha (derived from angle of attack) –X7	0	0.0396
delev (derived from elevon deflection) – X8	-0.0243	0.0264
Bflap (derived from body flap deflection) – X9	-0.3481	0.3333
Scenario 5: Mach 2 - Minimum angle of attack		
alpha (derived from angle of attack) –X7	-2.378	0.0396
delev (derived from elevon deflection) – X8	-0.4013	0.0264
Bflap (derived from body flap deflection) – X9	-5.0157	0.3333
Scenario 6: Mach 2 - Maximum angle of attack		
alpha (derived from angle of attack) –X7	2.378	0.0396
delev (derived from elevon deflection) – X8	-0.0149	0.0264
Bflap (derived from body flap deflection) – X9	10	0.3333
Scenario 7: Mach 10 - Nominal angle of attack		
alpha (derived from angle of attack) –X7	-0.7927	0.0396
delev (derived from elevon deflection) – X8	2.3476	0.0264
Bflap (derived from body flap deflection) – X9	6.4322	0.3333
Scenario 8: Mach 10 - Minimum angle of attack		
alpha (derived from angle of attack) –X7	-2.378	0.0396
delev (derived from elevon deflection) – X8	0.7927	0.0264
Bflap (derived from body flap deflection) – X9	20	0.3333
Scenario 9: Mach 10 - Maximum angle of attack		
alpha (derived from angle of attack) –X7	2.378	0.0396
delev (derived from elevon deflection) – X8	0.7927	0.0264
Bflap (derived from body flap deflection) – X9	20	0.3333

Limit States

A limit state defines the boundary between success and failure of the system. In this problem, each of the 9 scenarios has two limit states, one for the lower bound and one for the upper bound of C_m (pitching moment coefficient). The lower bound limit state is

$$g_{lower} = 0.01 + C_m, \quad (1)$$

and the upper bound limit state is

$$g_{upper} = 0.01 - C_m \quad (2)$$

The probability of failure is defined as

$$P_f = P(C_m < -0.01) + P(C_m > 0.01) = P(g_{lower} < 0) + P(g_{upper} < 0) \quad (3)$$

The probability of failure for each limit state is the volume integral under the joint probability density function of all the input variables over the failure region (i.e. where $g < 0$) as shown in Figure 2.

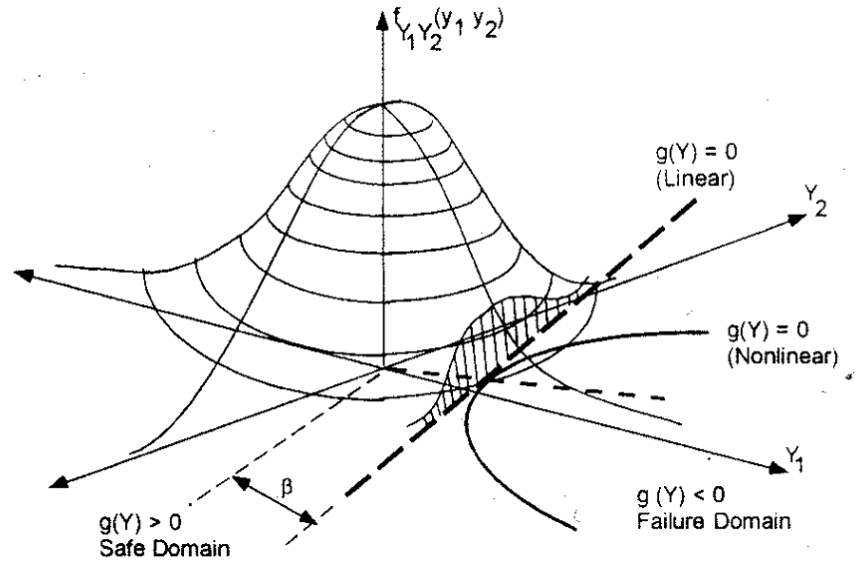


Figure 2: Limit State and Probability of Failure

First Order Reliability Method (FORM)

In this method, the minimum distance from the origin to the limit state is expressed as $\beta = \sqrt{(X^*)^T (X^*)}$, where X^* is the point of minimum distance from the origin to the limit state. The minimum distance point on the limit state is referred to as the most probable point (MPP). X is the vector of reduced normal variables. In this case, variables are assumed normal, but they can be transformed into equivalent normal variables for the FORM analysis if a different distribution is appropriate. A linear (first order) approximation of the volume integral (or failure probability) at the MPP is computed as $\Phi(-\beta)$, where Φ is the cumulative distribution function (CDF) for a standard normal parameter. A graphical representation is shown in Fig. 3.

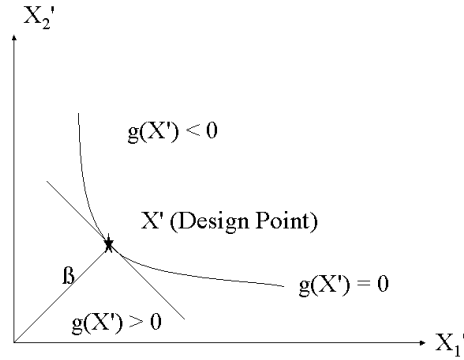


Figure 3: First-Order Reliability Method

The MPP is found iteratively using the Rackwitz- Fiessler formula (Haldar and Mahadevan, 2000).

$$X_{k+1}^* = \frac{1}{|\nabla g(X_k^*)|^2} [\nabla g(X_k^*)^T (X_k^* - g(X_k^*))] \nabla g(X_k^*), \quad (4)$$

Here X_{k+1}^* is the MPP at the $(k + 1)^{\text{th}}$ iteration, and $\nabla g(X_k^*)$ is the gradient vector (vector of derivatives of the limit state function with respect to each variable.)

The direction cosines (alphas) at the MPP, $\alpha_i = \frac{\frac{\partial g}{\partial x_i} * \sigma_{x_i}}{\sqrt{\sum (\frac{\partial g}{\partial x_i} * \sigma_{x_i})^2}}$ are a useful by-

product of the FORM method in that they describe the sensitivity of the limit state to each of the input parameters. The vector, α is the unit gradient of the limit state, g , in standard normal space, at the most probable point. This information is helpful on several accounts. Variables with low sensitivities can be approximated as deterministic to reduce the computational complexity of the problem. Also, reducing the standard deviation of variables with high sensitivities will achieve the greatest gains in system reliability.

Optimization

As stated earlier, the probabilistic optimization problem is formulated as follows:

Minimize mean weight such that the pitching moment coefficient for all 9 scenarios has a low probability (less than 0.1) of failing to be within acceptable bounds [-0.01, +0.01]. Optimization is done in the same manner as before using a gradient based non-linear optimizer. However, instead of constraining C_m to be between [-0.01, +0.01], the probability of exceeding the bounds is constrained to be less than some acceptable value. Also, the means of the input variables are the actual design variables that are altered during the optimization, and the mean weight is minimized.

Results and Discussion

Two deterministic and four probabilistic runs are performed and their resulting optimal dry weight (or mean dry weight) compared with one another. The deterministic runs were based on factors of safety of 1 and 1.5 and were formulated as follows:

$$\text{Minimize empty weight} \quad (5)$$

$$\text{Subject to } -0.01 \leq \frac{C_{m_i}}{\text{F.S.}} \leq 0.01, \text{ for } i = 1 \text{ to } 9 \text{ scenarios (from Table 5)}$$

After the optimal design was selected deterministically, a FORM probabilistic analysis was performed (assuming random variation of the design variables) to determine the probability that the pitching moment was within the standard bounds, $[-0.01, 0.01]$. The probabilistic runs were based on acceptable probabilities of failure of 5%, 10%, 15%, and 20%. In this case, the following formulation was used:

$$\text{Minimize mean empty weight} \quad (6)$$

$$\text{Subject to } P(-0.01 \leq C_{m_i} \leq 0.01) \geq P_f$$

Fig. 4 is a graphical comparison of the optimal weight for each analysis. The first deterministic run uses a factor of safety of 1 so that C_m was simply constrained to $[-0.01, +0.01]$ as stated in the original deterministic formulation. As would be expected, this method yields the lowest optimal weight. However, such a solution corresponds to a 50% probability that the pitching moment actually fails to be within these bounds. A post-optimization FORM analysis proves this but it is also evident by inspection. In effect, the deterministic optimization drives the mean of C_m to the boundary.

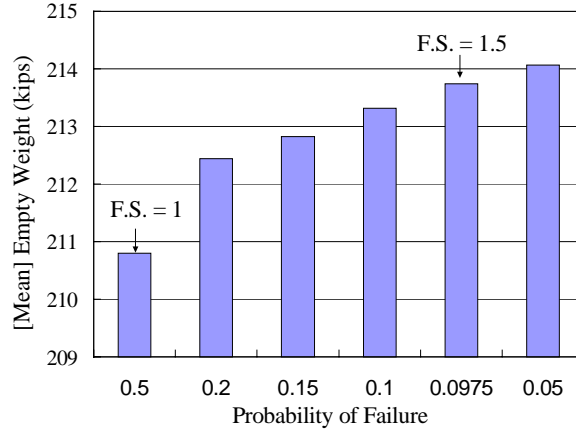


Figure 4: Weight Optimization Results

The second deterministic optimization with a factor of safety of 1.5 in effect reduces the pitching moment constraint to $[-0.667, +0.667]$. A post optimization probabilistic analysis shows that this method gives a 9.75% probability that the true C_m fails to be within the original bounds, $[-0.01, 0.01]$. The probabilistic optimization results show, as expected, that the optimal mean weight is inversely proportional to the required failure probability.

It is interesting to note that the 1.5 factor of safety compares closely with the 10% failure probability (P_f) case in both probability of failure (9.75% for the 1.5 FS vs. 10% P_f) and optimal mean empty weight (213.7 kips vs. 213.3 kips). Although the 1.5 FS approach and the 10% P_f approach both give similar designs, the disadvantage of the factor of safety approach is that it does not quantify the failure probability. Such information is gained only from probabilistic analysis. If we want to improve the probability of satisfying our original constraint by a specific amount, we cannot know beforehand what new factor of safety to use. Probabilistic optimization, though it requires more computation than deterministic optimization, is not difficult for simple

closed form analyses that use response surfaces. It enables the designer to consider input variability and in essence, directly ‘design for’ a reliability requirement.

A second graph, Fig. 5 shows the probabilistic sensitivity indices of the various input variables for three of the nine scenarios: the maximum angle of attack scenarios at each of the three speeds (scenarios 3, 6 and 9 from Table 4). The sensitivity indices are

the direction cosines, from the probabilistic analysis $\alpha_i = \frac{\frac{\partial g}{\partial x_i} * \sigma_{x_i}}{\sqrt{\sum (\frac{\partial g}{\partial x_i} * \sigma_{x_i})^2}}$, discussed

in a previous section. They are directly related to the derivatives of the pitching moment failure probability with respect to the input random variables, normalized so that the sum of the squares is one. Note that the probabilistic sensitivity indices combine physical sensitivities (derivatives of the limit state with respect to the design random variables) as well as the scatter (through the σ_{x_i} term) of the random variables. Such information is only available through probabilistic analysis. In addition, the probabilistic sensitivity indices give direct indication for the design improvement and optimization. These indices have been used to derive reliability-based design safety factors (Ang and Tang, 1984). Also, these indices help to identify the important variables affecting the system failure probability, and therefore facilitate decisions with respect to resource allocation for data collection, improved modeling, etc. On the other hand, variables with low sensitivity indices may be treated as deterministic, thus reduction the computation effort.

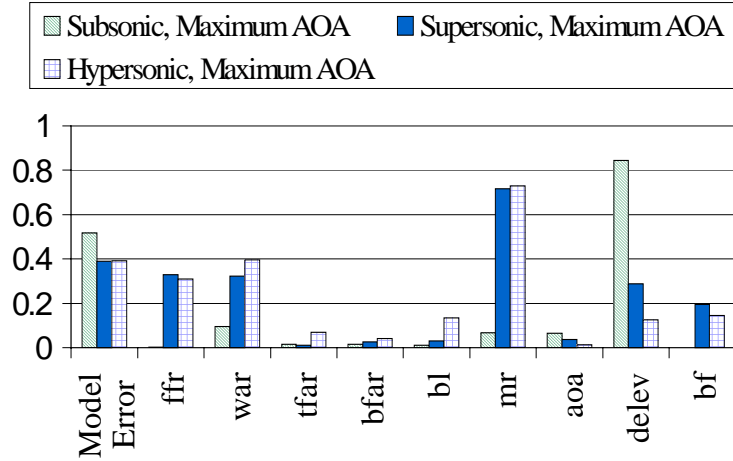


Figure 5: Variable Sensitivities to Pitching Moment

It is seen from Fig. 5 that the assumed model error has a significant effect for all scenarios but that the driving effect is mass ratio for the supersonic and hypersonic scenarios and elevon deflection for the subsonic scenario. Similar results are given for other scenarios with the same speeds. (For example, all three subsonic scenarios have high pitching moment sensitivity with respect to elevon deflection.)

Conclusions

This chapter has presented and applied a methodology for probabilistic multidisciplinary optimization. The application demonstrated involves codes from three disciplines and considers nine limit states all related to the pitching moment (C_m) constraint. The combination of response surfaces and first order reliability analysis provides a valuable tool for system conceptual design. This methodology can be achieved for any defined limit state and could be expanded to incorporate additional probabilistic constraints for more complex problems (e.g. for lift and drag failure in this

problem). Unlike simulation methods, FORM converges quickly even for very low probabilities of failure and is very conducive to optimization.

It is important to note that this is only the first step of the conceptual design. As design progresses, additional detail at the component level is required and higher fidelity analyses are needed for accuracy. The next chapter addresses a strategy for stepping down from a global design to a component level design (i.e. a liquid hydrogen tank). Chapter V deals with the additional complexity of higher fidelity multidisciplinary analysis.

As more disciplines are integrated into the conceptual design process, several different types of constraints may need to be included. Some of the limit states corresponding to these constraints may have a sequential relationship and may be linked through a probabilistic event tree in a system level analysis. This is the case for the liquid hydrogen tank design presented in the next chapter, where system failure may occur through a number of distinct failure modes.

CHAPTER III

LIQUID HYDROGEN TANK DESIGN

In the previous chapter, probabilistic methods were demonstrated for the geometry optimization of a reusable launch vehicle. This application was an example of an inter-disciplinary system-level design in that it considered a coupled analysis of geometry, weights, and aerodynamic disciplines. It was also a system-level design in terms of physical architecture, in that the geometry design variables define the global characteristics (length, radius, wing areas, etc.) of a vehicle comprised of many component parts (wings, fuel tanks, engines, etc.). At the same time, the probabilistic methodology applied was component-level reliability analysis since individual event probabilities (excessive pitching moment) were used as failure constraints. The work presented in this chapter expands on the global RLV design by considering the design for an individual fuel tank. This is a single disciplinary analysis, of a single physical element, analyzed in terms of multiple limit states.

Background

Global-Local Coupling in RLV System Design

Large engineering systems are too complex to study on a single level, so they must be decomposed for meaningful analysis and design. However, the system is not merely a collection of unrelated parts; the components of a system are coupled to one

another in a unique way as defined by the system architecture. Thus, a design process must be able to evaluate a system at the level of individual component performance and also at the level of the coupled system. To do this, engineers must manage both system decomposition as well as system re-integration. Typically, a system hierarchy is needed to do this.

In the design of complex systems, system hierarchy exists in several, related forms. For example, one form relates to the physical architecture of the system. Here the ‘system-level’ refers to the global physical design. This is contrasted with the ‘component-level’, which may include physical elements of the design (such as individual tanks, engines, etc.). Another hierarchy is along the lines of disciplinary analyses. Here, the ‘system’ is the integrated analysis involving several coupled disciplines (such as weights, aerodynamics, etc.). System level designs must be compatible across the individual ‘components’ or disciplinary analyses. Finally, the ‘system’ can refer to the sequence of events leading to a ‘system failure.’ In this sense, the individual ‘components’ are the single events in the failure sequence. Each event is defined by individual limit states. Mathematically, the system failure is represented by a series of unions and/or intersections of multiple limit states. These three forms of system architecture (disciplinary, physical, and failure) are depicted in Fig. 1 (a-c).

The engineering design process, if it is to result in an efficient system design, will be iterative in nature. Consider that a hierarchical top-down approach begins with a system-level or global design that is then broken into sub-systems, components, sub-components, etc. As designs are optimized for a given component, both the inputs and outputs are affected. These changes may affect coupled components on the same level as

well as the upstream (higher order) and downstream (lower order) levels (Buede, 2000). Iteration is required to obtain true system-level optimality as well as inter-level compatibility.

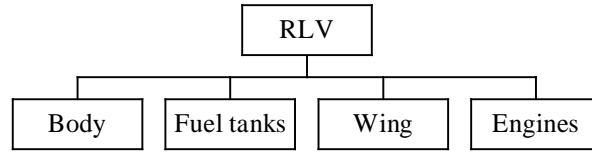


Figure 1(a) Physical Architecture

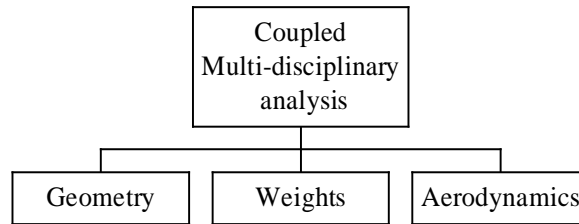


Figure 1(b) Disciplinary Architecture

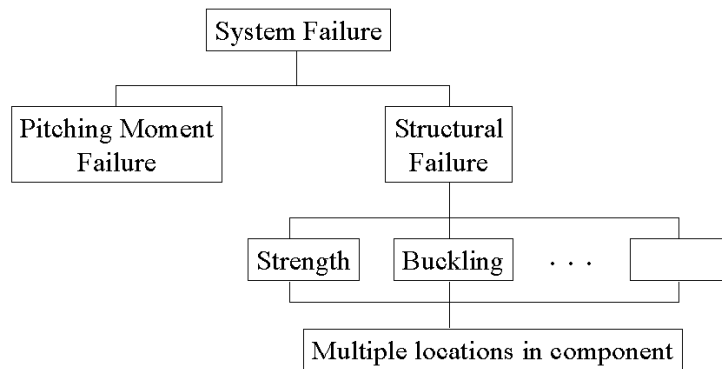


Figure 1(c) Failure Architecture

A launch vehicle is comprised of many components (Fig. 2). Each component must be designed to successfully perform its individual function, but must also integrate or 'fit' into the system as a whole. The subject of this chapter is a liquid hydrogen (LH_2) fuel tank. Design parameters include the shape, dimensions, and location of the tank as well as the material make-up of the tank walls. The primary operational requirement for the tank is that it must be large enough to hold the fuel necessary to complete the mission of the vehicle. At the same time, the tank must not occupy the same space as other components and must be strong enough to resist loading induced by the entire vehicle. The global system design drives both the fuel requirements (a function of the shape and weight of the vehicle) and the induced loading (a function of the weight distribution of the vehicle).

Of the three elements of tank design (location, shape, and wall material), this analysis focuses on the tank wall material. Trade studies have been conducted addressing optimal shapes and positioning for fuel tanks in the X-33 lifting body configuration (Dorsey, et al, 1999). However, stability considerations for the more basic slender body concept dictate that the two fuel tanks (liquid oxygen and LH_2), which comprise the bulk of the weight of the loaded vehicle, be at either end of the launch vehicle. In addition, the slender body vehicle concept lends itself to cylindrical tank geometry so that only the tank's length and radius are needed. Furthermore, the volume is obviously dictated by fuel requirements, and the tank radius is limited by the RLV geometry, both of which are products of the global design. For the scope of the following analysis, the LH_2 tank is assumed to be a typical cylindrical tank with given end eccentricity, located at a fixed distance from the end of the vehicle. With tank geometry and location driven by

requirements, the final considerations for design are the material and structure of the tank walls. In this respect, the tank should be sized such that it is as light as possible but strong enough to resist stresses induced by inertial loads.

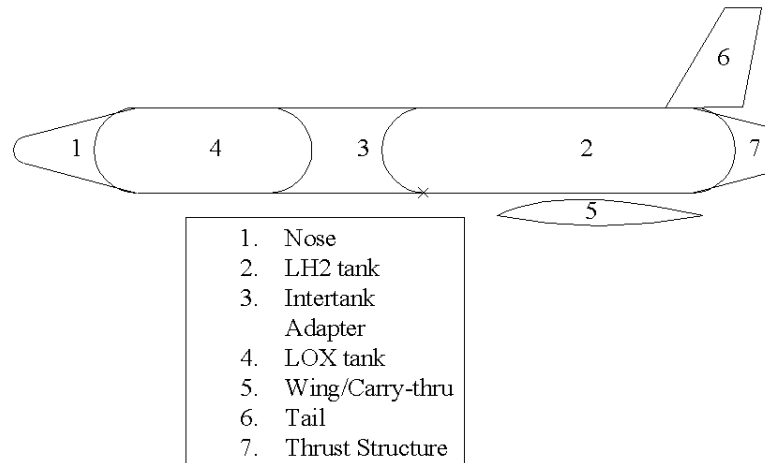


Figure 2: RLV Components

Structural Sizing Overview

Limit States

The goal of structural design in general can be described as achieving a structural capacity (through design) that is greater than the anticipated loading. This criterion is described by the basic performance function, $g = R - S$, where R is component capacity (or resistance) and S is the component load. The condition $g = 0$ is referred to as limit state. An alternate performance function is $g = \frac{R}{S} - 1$. In either case, when $g < 0$, the component is presumed to fail. Physics-based mathematical models predict R and S , and usually there are several modes of failure that must be considered. For example, under uniaxial loading the yield stress of material σ_y is the resistance and the applied stress σ

is the load, so that the failure limit state becomes: $\sigma = \sigma_y$. For multiaxial loading, the Tresca and Von Mises failure theories are competing models for failure of ductile materials. The Von Mises failure criterion for plane stress,

$$[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2] \leq \sigma_y^2 \text{ or} \quad (1)$$

may be written in limit state form:

$$g = \sigma_y^2 - [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2] \leq 0 \text{ or} \quad (2)$$

$$g = \sigma_y - [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} \leq 0 \quad (3)$$

Here, the component capacity, R , is represented by the yield stress, σ_y , and the equivalent Von Mises stress, $[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2}$ represents component loading, S (Popov, 1976).

Additional failure modes come into play when composite materials or various panel concepts (Fig. 3) are used for the tank wall structure. Examples include material strength, panel biaxial buckling, beam-column buckling, local buckling of flanges and webs, crippling of cross section, deformation, modal frequency, etc. Structural sizing software such as the commercially available *Hypersizer*TM, is useful for this assessment. *Hypersizer*TM computes limit states in the form of a margin of safety (M.S.) for a variety

of failure mode, where $M.S. = \frac{\text{ultimate failure load}}{\text{allowable failure load}} - 1$ (Collier, et al, 1999; Allen and

Haisler, 1985).

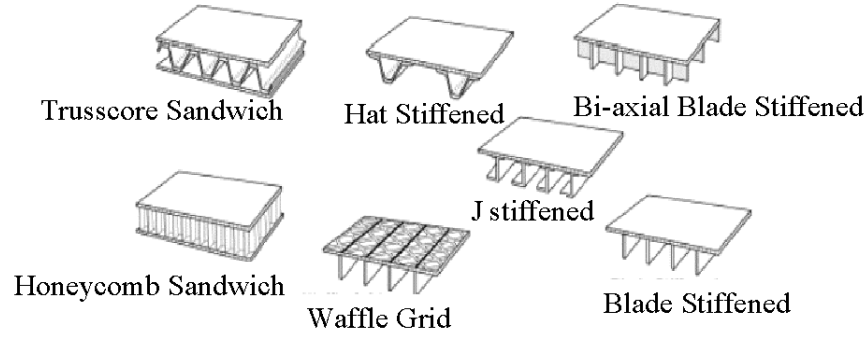


Figure 3: Panel Concepts

Accounting for Uncertainty

The limit states described above are model-based predictions of true performance, and they are associated with many uncertainties. Uncertainty exists for the material properties (which determine the system resistance) and the system loading. The limit states themselves also have inherent model uncertainty. For example, the Von Mises criterion is an approximate model of material failure under combined loading.

In traditional design, factors of safety are used to ‘over design’ the component in an attempt to accommodate these uncertainties (Lindberg, 1992). For example, in allowable stress design (ASD) for steel, $\sigma_a = \frac{\sigma_y}{\text{F.S.}} \geq \sigma_{\text{actual}}$, where σ_a is the allowable stress, σ_y is the yield stress, and F.S. is the factor of safety (typically 1.5 to 2.5). The load and resistance factor design method (LRFD) is gaining widespread acceptance over ASD methods, but it is also essentially a factor of safety approach. In LRFD, a capacity reduction factor, ϕ , is used to adjust the acceptable resistance (e.g. $M_n \geq \frac{M_u}{\phi}$, where M_n is the nominal moment strength and M_u is the ultimate or required strength), and load

factors are applied to design loads (e.g. $M_u = 1.4M_{\text{dead load}} + 1.7M_{\text{live load}}$). Both the ASD and LRFD approaches acknowledge that there is uncertainty in either the system loading (S), the system capacity (R), or both. Furthermore, LRFD associates varying degrees of uncertainty for different types of loading by using different load factors (for example, live loads are assumed to have a higher uncertainty than dead loads and so have a higher load factor). The appropriate factors have been codified for materials such as concrete and steel based on years of design experience with conventional structures giving a measure of system reliability that is implicitly understood by those in the industry. However, modern aerospace systems are forging new ground with respect to materials and operational environments. With these novel systems, there is no translation between factors of safety and reliability. Instead, program sponsors are likely to prefer reliability information in terms of the probabilities of failure.

The probability of failure gives a much clearer picture of system reliability. For one, it defines reliability in a way that is consistent regardless of design complexity, materials used, or loading environment. Another advantage is that the failure probability measure can be translated to failure costs. Unfortunately, failure probabilities are more difficult to assess than deterministic factors of safety. At the level of major components such as the LH₂ tank, it is not reasonable to perform the number of physical tests necessary to determine the tank's structural reliability experimentally. However, as with the RLV conceptual design, probabilistic methods may be used to assess the reliability based on uncertainty quantifications for the system resistance (i.e. material properties), system loading, and the physical model (system analysis followed by failure criterion evaluation).

There are multiple modes of failure for the RLV tank (i.e. Von Mises interaction failure, isotropic failure, panel buckling), multiple locations along the tank that could fail, and even multiple load cases (at various stages in the RLV's trajectory) that could cause failure. Each of these failure cases may be represented by a corresponding limit state. However, the overall reliability measure for the tank is the system failure probability, which synthesizes all of these modes. Since any single failure mode results in failure of the tank, a union operation mathematically represents system failure. There are several methods for approximating the probability of unions for dependent events. These include methods developed by Ditlevesen (1979), Hohenbichler and Rackwitz (1983), Gollwitzer and Rackwitz (1988), Madsen et al (1986), and Xiao and Mahadevan (1994).

Probabilistic Design of LH₂ Tank

Problem Definition

The design goal for the RLV tank is to minimize the weight of the tank while meeting the requirements for fuel capacity and structural integrity. The fuel capacity requirement is maintained by choosing the appropriate tank geometry. With tank geometry dictated by the global design, an optimization problem may be formulated to select the best design for the tank wall structure:

$$\text{Minimize Tank Weight} = f(R) \tag{4}$$

Subject to

$$R - S < 0 \text{ or } \frac{R}{S} - 1 \leq 0 \text{ \{for all failure modes\}}$$

where R is the tank resistance and S is the loading on the tank. The problem is re-formulated to consider uncertainties in R and S :

$$\text{Minimize } \mu_{\text{tank weight}} = f(f_R(R)) \approx f(\mu_R) \quad (5)$$

Subject to

$$P\left(\bigcup_{\text{all failure modes}} R - S\right) < P_{\text{required}}$$

This optimization formulation recognizes that the objective (tank weight) and constraints (failure limit states) are random variables. For well-defined optimization, objectives and constraints need to be selected from among the parameters that characterize the random distributions of these variables. In this case, the parameter mean tank weight is selected as the objective, and the probability of system failure is chosen as the constraint.

For isotropic panel concepts (such as a simple thin-walled tank), tank weight and resistance are perfectly correlated so that the design variable, μ_R , may replace the objective function. In this case, the reliability constraint is active and the designer may simply solve for μ_R to satisfy $P\left(\bigcup_{\text{all failure modes}} R - S\right) = P_{\text{required}}$.

Evaluating $P\left(\bigcup_{\text{all failure modes}} R - S\right)$ involves five subtasks: (1) defining an analytical

model for the system loading, S , based on information from the global analysis and the mission profile, (2) defining analytical models for various failure modes that incorporates the loading model and resistance, R , in terms of design variables (3) quantifying the uncertainty of inputs to the failure model, (4) using probabilistic methods to evaluate the failure probability of the individual limit states, and (5) using system probability methods to estimate the union of several limit states.

Calculating System Loads

Running load calculations are based on the methodology used in the C program, **RL** by Cerro (1996). (The running load is the applied stress times the thickness.) They are based on a simple beam model as depicted in Fig. 4. The model accounts for component weights as uniform distributed loads under axial and normal accelerations. In addition, fuel tanks experience ullage and head pressures. Reaction point locations (R1 and R2) represent lift, drag, wheel reactions, and/or the launching structure reactions depending on the mission stage (i.e. lift-off, ascent, landing, etc.). The calculations were programmed in *Matlab* (Mathworks, 2002) in order to simplify links with existing probabilistic analysis programs and optimization algorithms. They are summarized below:

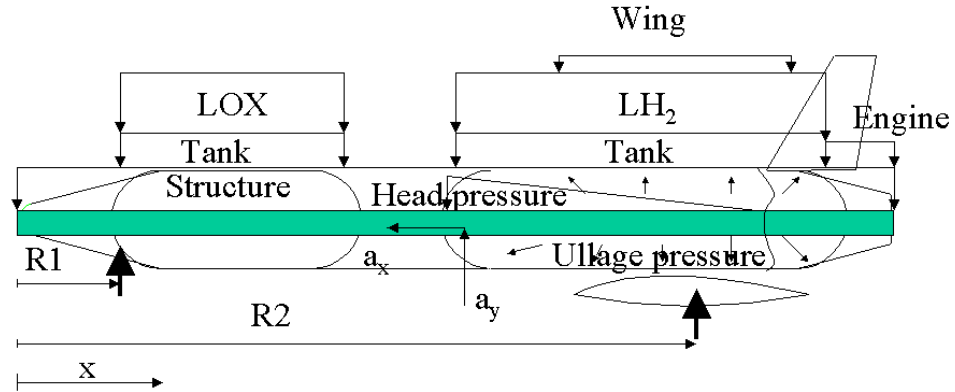


Figure 4: Simple Beam Model for RLV Loading

1. Ullage pressure:

$$N_x (\text{ullage}) = \frac{\text{ullage pressure} * \text{radius}}{2}, \quad (6)$$

$$N_y (\text{ullage}) = \text{ullage pressure} * \text{radius} \quad (7)$$

where the LH₂ ullage pressure is equal to 25 psi.

2. Head pressure:

$$N_x(\text{head}) = \frac{\text{head pressure} * \text{radius}}{2}, \quad (8)$$

$$N_y(\text{head}) = \text{head pressure} * \text{radius}$$

where,

$$\text{head pressure} = \text{head (inches)} * \text{fluid density} * \text{axial acceleration, and} \quad (9)$$

$$\text{head} = (\text{location of interest} - \text{tank start location}) * \text{percent fuel}/100;$$

and the fluid density for LH₂ is 4.421 lb/in³.

3. Axial force:

$$N_x(\text{axial}) = \frac{F}{2 * \pi * \text{radius}} \quad (10)$$

where F is the axial force calculated as

$$F = \sum w_i * \text{axial acceleration} \quad (11)$$

for all distributed weights, w_i up to the location of interest. Fuel weights are only added if the location is beyond the end of the tank.

4. Bending:

$$N_x(\text{bending}) = \frac{M * \text{radius}}{I} \quad \text{where} \quad (12)$$

I is the moment of inertia. M is the moment calculated based on shear and moment diagrams from simple beam theory for a set of uniformly distributed loads, w_i , applied at given beginning and end locations:

$$M = \int \int w dx \cong \sum_{i, b_i < x < e_i} w_i (x - b_i)^2 + \sum_{i, x > e_i} w_i (e_i - b_i)^2, \text{ and} \quad (13)$$

$$I = \pi * \text{radius}^3$$

5. Shear:

$$N_{xy}(\text{shear}) = \frac{V}{(\pi * \text{radius})} \quad (14)$$

where V is the shear calculated from the simple beam theory:

$$V = \int w dx \equiv \sum_{i, b_i < x < e_i} w_i (x - b_i) + \sum_{i, x > e_i} w_i (e_i - b_i), \text{ where } e_i \text{ and } b_i \text{ are the} \quad (15)$$

locations of the ending and beginning locations of the uniform distributed load.

Total running loads are found by adding the contributions from ullage, head, axial, bending and shear so that:

$$N_x = N_x(\text{ullage}) + N_x(\text{head}) + N_x(\text{axial}) + N_x(\text{bending}) \quad (16)$$

$$N_y = N_y(\text{ullage}) + N_y(\text{head}) \quad (17)$$

$$N_{xy} = N_{xy}(\text{shear}) \text{ \{only applies at side locations\}} \quad (18)$$

The simple beam model is a low fidelity analysis. It is useful for conceptual component design and updating the weight distribution of the global vehicle. However, higher fidelity models (based finite element analysis) would be needed to better predict component loads as design progresses to the next level.

System Resistance and Failure Limit States

There are multiple failure criteria (e.g. strength, panel buckling, etc.) that may be used to predict structural failure, and infinite locations on the component that may fail. Each of these cases is associated with a failure limit state. In addition, since loading

varies with the flight profile, there are an infinite number of failure limit states associated with flight conditions. System failure occurs if any part of the tank fails at any time during the mission based on any of the failure modes. Thus system failure may be modeled as a series or union of an infinite number of limit states. Obviously, only a finite number of failure limit states can be analyzed; this can be achieved by dividing the continuous spaces (e.g. tank location, flight profile) into finite regions. This concept is demonstrated in the following section through two models of system failure. The first model assumes a uniform, thin-walled tank and synthesizes failure at three different locations according to a single criterion, Von Mises failure. The second model assumes a segmented tank, and synthesizes three failure modes (i.e. failure according to three different criteria). In both models, three limit states are chosen for the sake of demonstration. Additional limit states would need to be considered in practice, adding computational expense without changing the basic methodology.

Von Mises failure of uniform, thin-walled tank at three locations: In an initial approach, the failure performance function ($g = R - S$) is determined by the Von Mises criterion. For thin, isotropic tank walls, the resistance is determined by the yield stress of the isotropic material and the thickness. In addition, stresses may be assumed to be uniform through the thickness for the thin-walled tank. Thus, it is useful to consolidate both the yield stress and thickness into a single design variable called the yield load ($N_{\text{yield}} = \sigma_y * \text{thickness}$). Using running loads in lieu of stresses, the Von Mises criterion may be written in the following form:

$$N_{\text{yield}} - \sqrt{\frac{(N_x - N_y)^2 + N_x^2 + N_y^2 + 6(N_{xy}^2)}{2}} \leq 0 \quad (19)$$

The running loads on a tank vary according to location (both axially and radially) as shown in Fig. 5. In addition, the loading on the tank varies according to flight conditions (e.g. different acceleration conditions and fuel volume). System failure is defined as failure at any location during any anticipated set of flight conditions. This analysis assumes a uniform, thin wall for the tank so that its geometry and material properties (and thus the limit load, N_{yield}) do not vary according to location. Here only one set of flight conditions is considered at three radial locations (0° , 90° , 180°) at a single axial location.

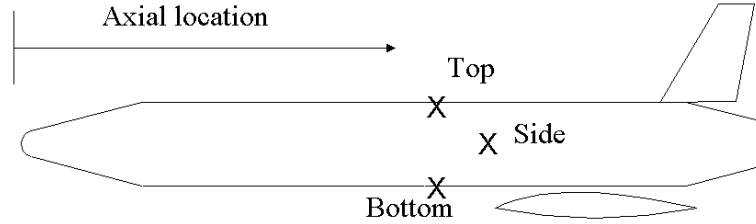


Figure 5: Three Axial Locations Analyzed for Failure

Multiple failure mode analysis of segmented, honeycomb tank panels: Since weight is of paramount importance, an isotropic, thin-walled tank is not the ideal design. Instead more complex concepts such as honeycomb sandwiches and blade-stiffened panels are more appropriate. In addition, an efficient wall design would have varying resistance properties according to the loading profile. This requires a more sophisticated approach for estimating tank resistance properties and analyzing failure modes. The material

management and structural sizing software, *Hypersizer*TM, is useful for this purpose (Collier, et al, 1999).

In this approach, the tank is divided into 10 sections along the length, and 4 sections along the circumference. Since the maximum load varies from panel to panel, they need not be identically designed. (However, the left and right side panels will be identical, requiring the design of 30 individual panels.) Loading on the panels is obtained from the simple beam model as before. In the analysis that follows, the side panel located close to the rear of the RLV is designed. *Hypersizer*TM includes a database of properties for a number of isotropic and composite materials, and it is able to consider several panel concepts. This demonstration utilizes a honeycomb sandwich concept consisting of top and bottom plates of Aluminum, AL2024 and Hexcell 1/8"-5052-.0015 for the sandwich material.

Design of the panels must specify the thickness of the plates and sandwich. Given the panel properties and applied loads, *Hypersizer*TM then evaluates the margins of safety for a number of failure criteria. For the tank walls, the significant failure modes are: isotropic strength in the transverse direction, Von Mises strength, and honeycomb buckling.

To facilitate probabilistic optimization, response surfaces for three failure modes (Von Mises, isotropic strength, and honeycomb buckling) were developed from a design of *Hypersizer*TM experiments. Equations for Von Mises and isotropic strength failure are known, so that the *Hypersizer*TM analysis is only needed to verify the effective yield stress for the panel material. They are given by Eqs. (3.20) and (3.21) respectively:

$$g_{VM} = \frac{84000 * t_{plate}}{\sqrt{N_x^2 + N_y^2 - N_x * N_y + 3N_{xy}^2}} - 1 \quad (20)$$

$$g_{Iso} = \frac{84000 * t_{plate}}{|N_y|} - 1 \quad (21)$$

A Box-Behnken design of experiments (Myers and Montgomery, 1995) was used to generate the 2nd order polynomial response surfaces for buckling of the honeycomb material. Based on the expected range of load variables (N_x , N_y , and N_{xy}), only the shear loading (N_{xy}), the plate thickness (t_{plate}) and honeycomb thickness (t_{hc}) affect the panel buckling failure margin. In addition, since failure is known to be inversely proportional to the applied loading, the polynomial response surfaces were based on $(1/N_{xy})$ and $(1/N_{xy})^2$ as well as $t_{plate/hc}$ and $t_{plate/hc}^2$. The response surface is given in Eq. 22.

(22)

$$g_{HCB} = .847 + .96x_1 + .986x_2 - .216x_3 + .077x_1^2 + .11x_2^2 + .007x_3^2 + .378x_1x_2 - .106x_1x_3 - .11x_2x_3$$

$$\text{where } x_1 = 4(t_{plate} - .075), \quad x_2 = 20(t_{hc} - .1), \quad \text{and } x_3 = -6000\left(\frac{1}{N_{xy}} + .003\right)$$

Variable Uncertainty

As seen in the structural analysis, the system loading is a function of several variables. The system resistance variable(s) constitute the design variable(s). For the thin-walled tank, this is reduced to a single parameter, N_{yield} . For the segmented tank, the honeycomb thickness (t_{hc}) is an additional resistance variable, and plate thickness is the design variable. All of these have a degree of uncertainty that affect the uncertainty of the calculated running loads. However, variables with an insignificant affect on the

output variability are assumed to be deterministic (i.e. constant). The variables are summarized in Table 1 below:

Table 1: Input Variables

Variables for Structural Tank Design						
	Parameter	Distribution	Origin	Mean	Cov	Description
$x1$	$R1$	Lognormal	mission	350	0.1	Location of first reaction point
$x2$	$R2$	normal	mission	2000	0.1	Location of second reaction point
$x3$	%_fuel	normal	mission	0.9	0.1	Percent of fuel remaining in tank
$x4$	ax	normal	mission	1	0.1	axial acceleration
$x5$	ay	normal	mission	1	0.1	normal acceleration
$x6$	mixratio	normal	mission	0.2	0.1	ratio of lox weight to lh2 weight
$x7$	radius	normal	global	377.4	0.0035	RLV & tank radius
$x8$	fuel wt	normal	global	2020000	0.0095	total fuel weight (lh2 and lox)
$x9$	N_{yield}	normal	local	design var	0.1	yield load of tank structure
	t_{plate}	normal	local	design var	0.1	top and bottom plate thickness
	t_{hc}	normal	local	0.1	0.1	honeycomb sandwich thickness
$c1$	oal	deterministic	global	2340		overall length
$c2$	wstruct	deterministic	global	109800		distributed load along entire RLV
$c3$	wwing	deterministic	global	21950		distributed load along wing
$c4$	wengine	deterministic	global	87810		distributed load along engine

The first 6 variables in Table 1 are determined by the mission profile for the launch vehicle. They will vary along the flight trajectory. The global variables are obtained from a Monte Carlo analysis of the RLV global design response surfaces as summarized below:

$$x7: \text{Radius} = \text{Fuselage Fineness Ratio} * oal \quad (23)$$

Fuselage fineness (fr) is an optimization variable for the global design. Response surfaces for overall length ($c1: oal$) were available from the weights design of experiments discussed in the last chapter.

$$x8: \text{Fuel Weight} = \text{Gross lift-off weight} - \text{empty weight} - \text{cargo weight} \quad (24)$$

Response surfaces for gross lift-off weight and empty weight come from the weights design of experiments. The cargo weight is 25,000 lb as given by mission

requirements. Unfortunately, the response surfaces do not give enough detail on component weights to develop the weight distribution needed for the tank structural analysis. Instead, the parametric estimates below were based on the weight distributions for the structure, wing, and engine of legacy vehicles. As components are designed in detail, this information may be updated to refine the structural analysis.

$$c2: W_{\text{structure}} = 0.5 * \text{empty weight} \quad (25)$$

$$c3: W_{\text{wing}} = 0.1 * \text{empty weight}$$

$$c4: W_{\text{engine}} = 0.4 * \text{empty weight}$$

Single Limit State Analysis - First Order Reliability Method

For the uniform thin-walled tank analysis, the limit state for tank failure at a single location is given by Eq. 19, which is a function of random input variables. Variability in the input parameters described above will result in some variability in the response, g . This variability in the response gives a probability of component failure, $P_f = P(g < 0)$. There are many methods for estimating the variability in response functions of input parameters with known variability. Here the first order reliability method (FORM) is chosen since it is a computationally inexpensive method effective for responses with low failure probabilities. FORM was discussed in detail in the previous chapter.

For the segmented, honeycomb panel tank, a two part probabilistic analysis is used. First, ten thousand Monte Carlo runs were performed (by varying the input variables from Table 1) on the loading analysis (Eqs. 6-18) to find the mean and standard deviations of the loads (N_x , N_y , and N_{xy}). The results are shown in Table 2. (Panel #'s

refer to longitudinal location. The qualifiers ‘top,’ ‘side,’ and ‘bottom’ refer to the circumferential location. N_y is the same for all circumferential locations. N_{xy} is zero for top and bottom panels. The #10 side panel is the subject of the design discussed herein.) The Kolmogorov-Smirnov test was conducted to assess conformity to a normal distribution type (Haldar and Mahadevan, 2000). All panel loads followed a normal distribution within a 10% significance level, and most were within 5% significance. FORM was then performed on the response surfaces of select panels considering N_x , N_y , and N_{xy} as random variables. The plate thickness and honeycomb thickness were also considered as random design variables. The advantage to this two-step approach is that it reduces the number of random variables. Although explicit gradients could be found for the response surfaces they would not be available for the true *Hypersizer*TM failure mode analysis; in this case, an additional finite difference run would be required for each random variable in order to use the Rackwitz-Fiessler algorithm for FORM. Table 2:

Table 2: Results of Monte Carlo Analysis of Loads

Panel #	1	2	3	4	5	6	7	8	9	10
Mean Values										
$N_x(\text{top})$	-662	-734	-746	-699	-593	-437	-218	73	436	871
$N_x(\text{side})$	562	512	461	411	360	297	226	155	84	13
$N_x(\text{bottom})$	1787	1758	1669	1521	1313	1031	670	237	-268	-845
N_y	4751	4751	4751	4751	4751	4751	4751	4751	4751	4751
$N_{xy}(\text{side})$	64	-11	-86	-161	-236	-321	-411	-502	-593	-684
Standard Deviations										
$N_x(\text{top})$	91	99	105	108	110	110	110	108	105	103
$N_x(\text{side})$	52	52	53	54	55	54	56	57	59	60
$N_x(\text{bottom})$	46	51	56	61	67	72	79	87	97	111
N_y	48	48	48	48	48	48	48	48	48	48
$N_{xy}(\text{side})$	11	10	9	9	9	9	9	9	10	11

Synthesizing Multiple Limit States for System Failure Estimation

In the uniform tank analysis, three failure locations are considered: top, side, and bottom (or 0°, 90°, 180° as in figure 5). Since system failure is defined when the tank fails at any one location, system failure is defined as a union of these three events:

$$P_{fs} = P\{(g_{top} < 0) \cup (g_{side} < 0) \cup (g_{bottom} < 0)\}. \quad (26)$$

or equivalently,

$$P_{fs} = 1 - P\{\overline{(g_{top} < 0)} \cup \overline{(g_{side} < 0)} \cup \overline{(g_{bottom} < 0)}\} \quad (27)$$

where $\overline{E} \equiv \text{not}(E)$.

Then, from DeMorgan's Rule,

$$P_{fs} = 1 - P\{\overline{(g_{top} < 0)} \cap \overline{(g_{side} < 0)} \cap \overline{(g_{bottom} < 0)}\} \quad (28)$$

or

$$P_{fs} = 1 - P\{(g_{top} > 0) \cap (g_{side} > 0) \cap (g_{bottom} > 0)\} \quad (29)$$

where $\cap \equiv$ intersection.

For the segmented, honeycomb panel analysis, three failure modes (of the #10 side panel) are considered for demonstration. They are honeycomb buckling, strength failure (Von Mises), and the transverse isotropic strength.

There are several methods of approximating the intersection of several events. In this case, the Hohenbichler approximation (Hohenbichler and Rackwitz, 1983) is used. This method uses the reliability indices, β , from the individual limit states and a correlation matrix, ρ . The correlation matrix is defined as: $\rho_{ij} = \alpha_i \bullet \alpha_j$, where

α_i and α_j are the direction cosine vectors such that $\alpha_i = \frac{\nabla g_i}{|\nabla g_i|}$. The Hohenbichler methodology will not be explained in detail here. However, the basic premise is to use an orthogonalization procedure to transform a set of limit states to a set of statistically independent limit states. Then, the probability of failure calculation is simply:

$$P_{fs} = P_{f1} * P_{f2}^* * P_{f3}^* \approx \Phi(\beta_1) * \Phi(\beta_2^*) * \Phi(\beta_3^*), \quad (30)$$

where β_2^* and β_3^* are the reliability indices for the transformed (i.e. statistically independent) limit states.

Results

The problem statement may now be refined as stated below:

$$\text{Minimize } \mu_{N_s} \quad (31)$$

Subject to

$$P_{fs} = P\{(g_1 < 0) \cup (g_2 < 0) \cup (g_3 < 0)\} \leq P_{acceptable}$$

(for Von Mises failure of uniform, thin-walled tank at three location)

$$\text{where } g_i = N_{yi} - \sqrt{\frac{(N_{xi} - N_{yi})^2 + N_{xi}^2 + N_{yi}^2 + 6(N_{xyi}^2)}{2}} \quad \text{or}$$

$$\text{Minimize } \mu_{t_{plate}}$$

Subject to

$$P_{fs} = P\{(g_{VM} < 0) \cup (g_{Iso} < 0) \cup (g_{HCB} < 0)\} \leq P_{acceptable}$$

(for 3-mode failure of segmented honeycomb panel, #10 side)

The optimization was performed for the thin-walled tank analysis using several different values for $P_{\text{acceptable}}$ using a bisection method. The results are depicted graphically in figure 6. (As a point of reference, a tank constructed from 60 ksi steel with a wall thickness of 0.25 inches has a yield load of 15,000 lb.) A margin of safety was also calculated for each of the three limit states. The margin of safety and the probability of failure for the side location are given in figure 7. The margin of safety is inversely proportional to the probability of failure, as one would expect.

Similarly, the optimization was performed for the segmented honeycomb tank. The optimal plate thickness is plotted against the acceptable system failure probabilities in Figure 8. In addition, the relative contribution of each of the limit states is depicted in Figure 9. It can be seen from this graph that the strength (Von Mises) and isotropic failure modes dominate the design requirements for this panel.

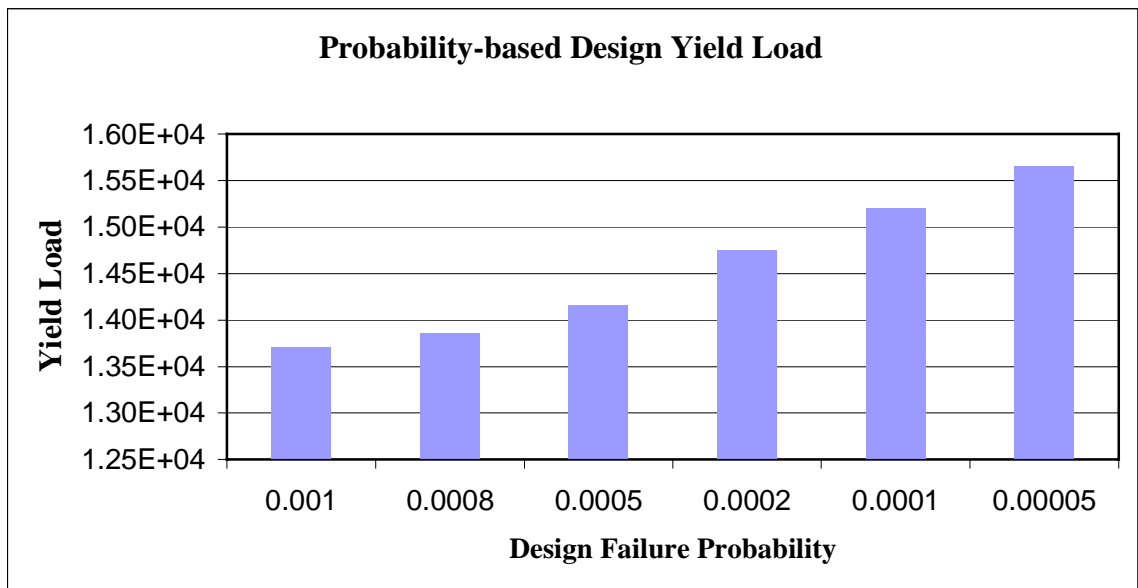


Figure 6: Design Failure Probability

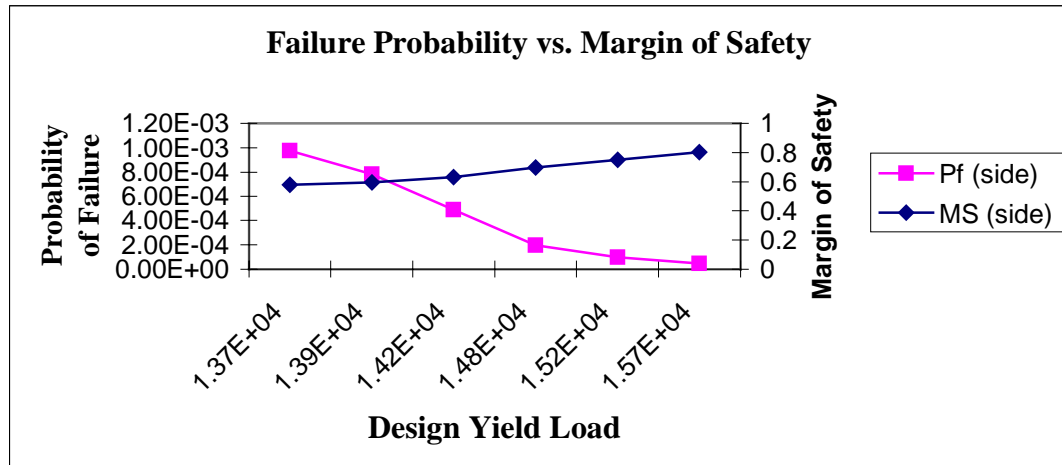


Figure 7: Design Yield Load

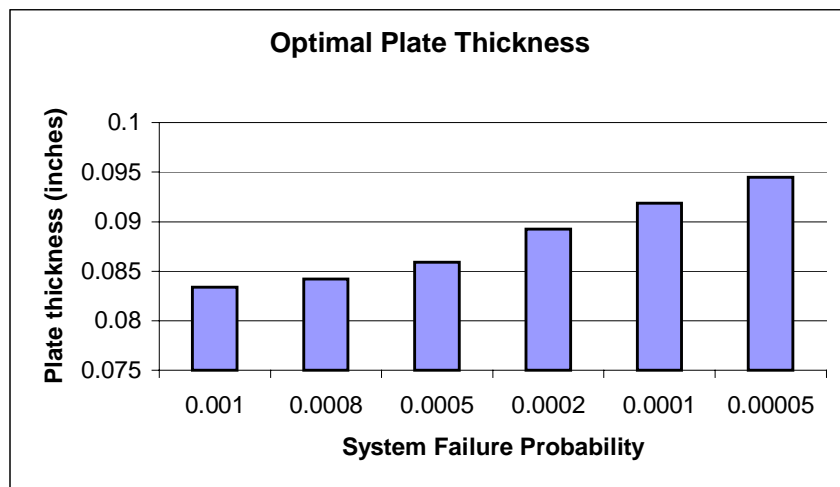


Figure 8: Optimal Plate Thickness

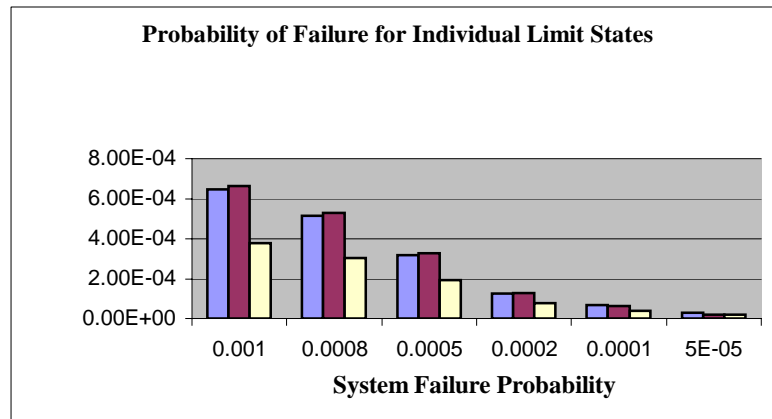


Figure 9: Probability of Failure for Individual Limit States

Conclusions

This application demonstrates the flow of probabilistic design information from a global, low-fidelity multi-disciplinary analysis to a higher fidelity, local, single discipline analysis. Geometric variables (fuselage fineness, overall length), established through the global design in the previous chapter, established the tank radius (Eq. 23). In addition, the vehicle empty weight and gross lift-off weights, outputs from the global weight optimization, were used to determine the vehicle loading distribution (Eqs. 24 and 25) for the local tank sizing analysis.

The application described is limited in that it only depicts a first iteration of coupling in a single direction. In other words, the local analysis of the RLV tank refines or updates the information on the global vehicle. This new information needs to be re-analyzed in the context of the global multidisciplinary design, which in turn updates information at the local level. This process of passing information between global and

local levels continues until acceptable convergence is reached. For the RLV design, global weight estimates from the low-fidelity global analysis are replaced by detailed component information after they are designed for structural reliability. This affects other disciplines such as aerodynamics and propulsion at the global level. These coupled disciplines affect the fuel requirements and the vehicle loading, and thus the local structural analysis must also be updated.

In addition to the global to local coupling, this application demonstrates a probabilistic optimization formulation for system failure of multiple limit states. For an RLV fuel tank, structural failure is subject to many possible failure modes, several locations, and a load history that varies according to the stage of flight. In this analysis, only three modes are considered for demonstration. In reality, an infinite number of failure limit states are applicable. One practical approach to solving such a problem is to increment the continuous fields for which failure modes apply. Then, a technique such as branch and bound may be appropriate to identify the failure modes of most significance before system reliability methods are to be applied.

The analysis presented in this chapter, like that in the previous chapter is of fairly low fidelity. These computationally efficient models are effective for probabilistic analysis and optimization where several iterations are necessary. However, as more design detail is required, higher fidelity models are needed. In this case, the simple beam load model and *Hypersizer*TM response surfaces will not be adequate. In the Chapter V, integration issues arising from higher fidelity analyses in multidisciplinary systems are addressed through a hypersonic missile application.

CHAPTER IV

COUPLED SYSTEMS

In the second chapter, probabilistic optimization was performed for the conceptual design of a reusable launch vehicle. The individual disciplinary analyses were integrated through response surfaces, which enabled efficient probabilistic analysis and optimization. However, the response surface approach does not have the precision needed for design beyond the earliest conceptual stage, when additional detail is required. The third chapter explored a single-discipline analysis of a component, and demonstrated the information flow from a low fidelity global model to a higher fidelity local analysis, but did not address the re-integration of the higher fidelity design at the system level. To complete system design, a multidisciplinary framework must allow coordination or re-integration of the disciplinary efforts. It is within this framework that probabilistic analysis and optimization tools must be applied in order to achieve improvements and meet design goals for cost and risk. Unfortunately, multidisciplinary integration can be complex and computationally intensive, a problem that is exacerbated when iterative probabilistic analysis and optimization tools are applied.

One challenge that surfaces for integrated multidisciplinary systems involves feedback coupling. In this case, iterative convergence loops are needed to resolve inconsistencies in feedback state variables. In a traditional ‘black box’ or fully coupled approach, any multidisciplinary analysis (MDA) convergence loops are nested inside loops for probabilistic analysis and/or optimization. The resulting computational effort

is unacceptable for most higher fidelity analyses. This chapter explores various alternatives to the “black box” approach for applying probabilistic analysis to multidisciplinary systems with feedback. These techniques are applied to a simple mathematical model and compared to one another with respect to accuracy and computational effort.

The Nature of Probabilistic Multidisciplinary Optimization

Multidisciplinary Analysis

Multi-disciplinary analysis (MDA) involves integrating individual (or discipline specific) analyses, which are *coupled* to one another through shared input and output data. This coupling is represented by state variables, which are variables derived as output from one discipline but required as input to other disciplines. MDA is characterized by a compatibility requirement; in other words, state variables must be consistent across all disciplines.

For analyses performed in a particular sequence, interdisciplinary coupling may be either a feed-forward or feedback type. For feed-forward coupling, the output of an earlier analysis feeds ‘forward’ as the input of a later analysis. Alternately, feedback occurs when a coupled analysis must be performed prior to the analysis that determines its input. Convergence loops are required to resolve feedback, requiring multiple ‘runs’ of a single set of analyses. Intelligent sequencing may minimize feedback coupling, but may not be able to eliminate it entirely (Browning, 2001). Even strictly feed-forward

MDA systems can be computationally expensive since performing analyses in sequence prevents the timesaving approach of parallel computing.

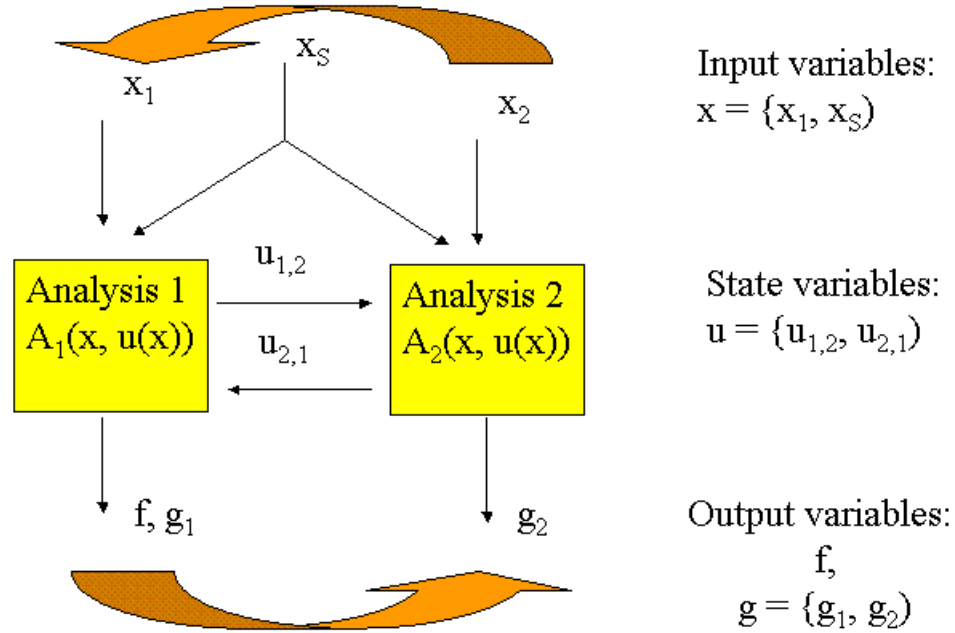


Figure 1: Feedback Coupling of a Two-Discipline System

Figure 1 depicts coupling in a two-discipline system. Here x_1 and x_2 represent local input variables to analyses 1 and 2 respectively, while x_s indicates input variables common to multiple analyses. Variables $u_{1,2}$ and $u_{2,1}$ are the state variables (such that $u_{i,j}$ is an output of analysis i and an input to analysis j). The system output variables are f, g_1 , and g_2 ; in the context of optimization, f may represent a system objective and g may represent system constraints. It may be seen that, regardless of which analysis is performed first, an unknown input state variable (either $u_{1,2}$ or $u_{2,1}$) is needed. Systems with feedback coupling are typically solved with fixed-point iteration. In other words, assumed values for the unknown state variables are initially used; then they are updated

by performing the analyses from which they are derived; the analyses are performed again with the updated values; this process continues until convergence is reached.

Using fixed-point iteration within probabilistic optimization algorithms is not usually an ideal approach. For one, the computational effort of a single set of the disciplinary analyses is multiplied by the number of fixed-point iterations, then by the number of probabilistic analysis loops, and finally by the number of optimization loops. This effect is depicted in Figure 2.

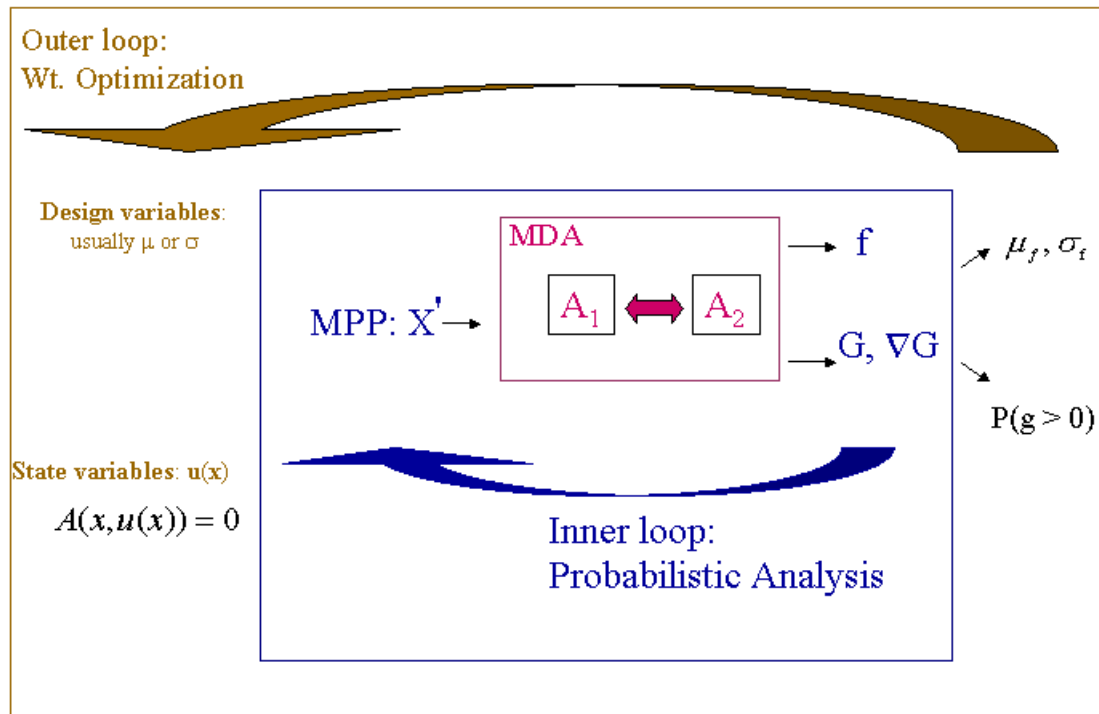


Figure 2: Probabilistic Optimization for Systems with Feedback

Another difficulty is in obtaining gradient information, usually required for the more efficient probabilistic analysis and optimization algorithms. If a finite difference method were used, the convergence process would need to be repeated for each variable.

Furthermore, one might select a less stringent convergence criterion to avoid unnecessary fixed-point iterations, but this introduces ‘noise’ that can interfere with finite difference estimates for the gradient. Thus, proven probabilistic analysis or optimization algorithms may not converge for these systems. For example, Mahadevan and Gantt (1998) show that a traditional probabilistic optimization approach for a coupled electronic packaging system does not converge for many starting points and requires over 10,000 function evaluations for those starting points that do lead to convergence.

Multidisciplinary Optimization (MDO)

The standard optimization problem is formulated as follows:

$$\begin{aligned} &\text{Minimize } f(\mathbf{x}) \\ &\text{Subject to } g(\mathbf{x}) = 0 \text{ and } h(\mathbf{x}) \geq 0 \end{aligned} \tag{1}$$

where \mathbf{x} is a vector of design variables, $f(\mathbf{x})$ is the objective function, $g(\mathbf{x})$ are the equality constraints, and $h(\mathbf{x})$ are inequality constraints. Both gradient-based and non gradient-based non-linear programming algorithms are available to solve problems of this type. Obviously, gradient-based methods require at a minimum that the objective function be differentiable in the first order. When gradients cannot be obtained directly, approximation methods such as finite differencing are often used.

In a multi-disciplinary context, the optimization problem is expanded to include state variables, $u(\mathbf{x})$, representing the results of intermediate, disciplinary analyses:

$$\begin{aligned} &\text{Minimize } f(\mathbf{x}, u(\mathbf{x})) \\ &\text{Subject to } h(\mathbf{x}, u(\mathbf{x})) = 0 \text{ and } g(\mathbf{x}, u(\mathbf{x})) \geq 0 \end{aligned} \tag{2}$$

The state variables must satisfy the multi-disciplinary compatibility requirement, often written in matrix form:

$$A_i(\mathbf{x}, u(\mathbf{x})) = 0, \text{ for each } i = 1, \dots, \text{number of disciplines in system} \quad (3)$$

A direct approach is to reduce the MDO formulation to the standard optimization problem through variable reduction. In other words, the only independent optimization variables are the design variables, \mathbf{x} , and the disciplinary state variables, $u(\mathbf{x})$ must be solved for at every iteration in the system optimization. This is also known as the Multidisciplinary Feasible Method (MDF). The limitation to this approach is that it can be computationally expensive. A reduction in the overall computation time may be accomplished if disciplinary analyses can be done in parallel. This approach requires an MDO problem formulation that decouples the disciplinary analyses from the multidisciplinary system optimization and from one another. Methods that use this tactic vary in the way they achieve disciplinary autonomy (a decoupling mechanism) and ensure multidisciplinary feasibility (a re-coupling mechanism). In the Individual Discipline Feasible (IDF) method, for example, auxiliary variables representing interdisciplinary flow are used to achieve autonomy for disciplinary analyses. Multidisciplinary feasibility is maintained in the system optimization through compatibility constraints that must be satisfied at the final solution. Since disciplinary autonomy is not maintained through the optimization (only for analysis), computational difficulties in integration often remain. In another method, collaborative optimization (CO), disciplinary feasibility is achieved through disciplinary optimization problems and system-level compatibility constraints. Here, system-level design variables (ξ, v) are

decoupled from disciplinary design variables (\mathbf{x} , $u(\mathbf{x})$). Compatibility constraints are added that require the sum of square of the errors to be zero.

$$\text{Minimize } f(\boldsymbol{\xi}, \mathbf{v}) \quad (4)$$

$$\text{Subject to } C(\boldsymbol{\xi}, \mathbf{v}) = 0,$$

where $C(\boldsymbol{\xi}, \mathbf{v})$ is a system of constraints of the form, $c_i = \frac{1}{2} [\|\boldsymbol{\xi}_i - x_i\|^2 + \|\mathbf{v}_i - u(x_i)\|^2]$

Separate disciplinary optimizations are performed to minimize the compatibility condition (c_i). Constraints other than the compatibility constraints may be given at the disciplinary level (Alexandrov and Kodiyalam, 1998).

Both IDF and CO have practical limitations for complex problems due to the nature of the interdisciplinary coupling formulations. Bi-level Integrated System Synthesis (BLISS) is a method developed and refined at NASA (Sobiesczanski-Sobieski et al, 2000), which uses sensitivity derivatives to reduce the complexity of interdisciplinary coupling. Like CO, optimization is performed at both disciplinary and system levels. System-level derivatives are calculated at a given design point; these derivatives define a linear model of the system objective function which becomes the objective function for the disciplinary optimizations. BLISS is particularly useful in that it is a modular formulation that can integrate disciplinary analysis tools at different fidelity levels appropriate to different design phases.

Probabilistic Analysis and Optimization

Alternatives for formulating probabilistic optimization problems are discussed in the first chapter. For example, both the global RLV design and the tank sizing applications have the following form:

$$\text{Minimize } \mu_F(f_x(x)) \quad (5)$$

subject to

$$P[g(f_x(x)) \leq 0] \leq P_f$$

where $f_x(x)$ is the probability density function for the design variables, μ_F is the mean of the objective function, $g(f_x(x)) \leq 0$ signifies failure to meet the constraint, and P_f is the acceptable (or design) failure probability. An first-order second moment (FOSM) method was used to estimate μ_F and the Rackwitz-Fiessler First-order Reliability Method (FORM) was selected to evaluate $P[g(f_x(x)) \leq 0]$.

Like optimization, probabilistic analysis is usually an iterative process, which multiplies the cost of a single multidisciplinary analysis. For probabilistic optimization, a nested analysis is generally required with probabilistic analysis in an inner loop and optimization in an outer loop. For example, to solve the formulation given in equation (5), the Rackwitz-Fiessler Newton step is applied iteratively each time the optimization algorithm calls for an evaluation of the constraint. This compounds the problem of computational expense. However, if commonalities between optimization and probabilistic analysis can be exploited, there is hope to streamline probabilistic MDO.

To understand these commonalities, it is useful to review the premise behind the first order reliability method (FORM). It is essentially an optimization problem where the objective function is the reliability index β , which is the minimum distance from the origin to the limit state in standard normal space. The point on the limit state at which this minimum β occurs is referred to as the most probable point or MPP. (As a reminder, a graphical representation of the FORM concept is reproduced in Fig. 3).

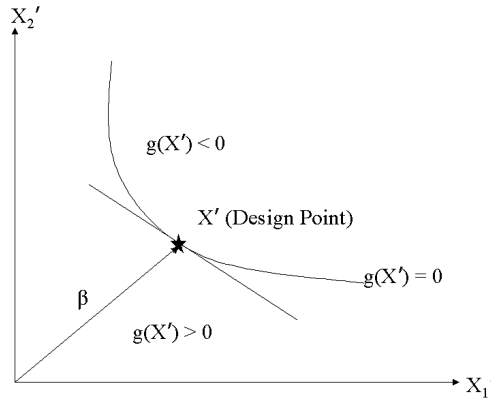


Figure 3. First - Order Reliability Method

Finding the most probable point is expressed as an optimization problem:

$$\text{Minimize } \beta = \sqrt{(X^*)^T (X^*)} \quad (6)$$

Subject to

$$g(X') = 0$$

In the applications of the previous two chapters, the Rackwitz-Fiessler Newton-type formula (Eq. 4 in Chapter II) was used iteratively to find the MPP. Other optimization algorithms such as sequential quadratic programming (SQP) may be used instead. Furthermore, alternate formulations for multidisciplinary systems like those used in many MDO methods could prove helpful.

Decoupling: A Strategy for Multidisciplinary Systems

Decoupling is a basic tactic for handling multidisciplinary analysis, and it is the basis for many of the MDO methods such as those discussed earlier in the chapter (e.g. IDF, CO, BLISS, etc.) Consider, for example the traditional MDO formulation:

$$\text{Minimize } f(\mathbf{x}, u(\mathbf{x})) \quad (7)$$

$$\text{Subject to } h(\mathbf{x}, u(\mathbf{x})) = 0 \text{ and } g(\mathbf{x}, u(\mathbf{x})) \geq 0$$

and the Individual Feasible Method (IDF) formulation:

$$\text{Minimize } f(\mathbf{x}, u) \quad (8)$$

$$\text{Subject to } h(\mathbf{x}, u) = 0, g(\mathbf{x}, u) \geq 0 \text{ and } u(\mathbf{x}) = u$$

In the IDF approach, decoupled auxiliary variables, u , replace the coupled state variables, $u(\mathbf{x})$. The additional constraint, $u(\mathbf{x}) = u$, enforces system compatibility at the optimal solution but not necessarily at each iteration. From a probabilistic perspective, using auxiliary variables raises some interesting questions. For one, state variables that are dependent on random input variables will obviously be random variables themselves, but how does one select an appropriate probability density function (pdf)? Also, if competing pdf's are obtained from each discipline, what is the appropriate compatibility requirement? Finally, if assumptions are made regarding pdf's for the auxiliary variables, how will any errors propagate to the system output variables? In other words, can the result be trusted?

In the sections that follow, three probabilistic analysis techniques for feedback systems are given. The first technique was presented by Du and Chen (2002) and uses the IDF MDO formulation in applying the first-order reliability method. The second technique uses a first-order second moment (FOSM) method to characterize state variables. The final technique uses the idea of Gibbs sampling in order to apply Monte Carlo probabilistic analysis to the decoupled system. Each of these techniques is applied to a mathematical two-discipline example system. In addition, as a baseline, the basic Monte Carlo method and FORM (using both sequential quadratic programming and the

Rackwitz-Fiessler Newton-type formula of Eq. 4 in Chapter II) are applied to the same system without decoupling. The results are compared with respect to accuracy and computational efficiency.

Mathematical Example: 2-Discipline System

This example is a two-discipline system as depicted in Fig. 1. It was taken from Du and Chen (2002). The functional relationships are as follows below.

Analysis 1	Analysis 2
$\mathbf{x}_s = \{x_1\}, \mathbf{x}_1 = \{x_2, x_3\}$	$\mathbf{x}_s = \{x_1\}, \mathbf{x}_2 = \{x_4, x_5\}$
$u_{1,2} = x_1^2 + 2x_2 - x_3 + 2\sqrt{u_{2,1}}$	$u_{2,1} = x_1 x_4 + x_4^2 + x_5 + u_{1,2}$
$g_1 = 4.5 - (x_1^2 + 2x_2 + x_3 + x_2 e^{-u_{2,1}})$	$g_2 = \sqrt{x_1} + x_4 + x_5(0.4x_1)$
$f = N/A$	

The limit state for the system is given by g_1 , so that the probability of failure, P_f , is given by $P_f = P(g_1 < 0)$.

Although this coupled system may be solved algebraically by variable reduction, the applications are all based on using fixed-point iteration for feedback convergence. (In other words, a trial value of $u_{2,1}$ is selected, $u_{1,2}$ is computed from analysis 1, then $u_{2,1}$ is computed from analysis 2; the process is repeated with the new value for $u_{2,1}$ until it converges.) This is done to simulate the behavior of real multidisciplinary systems that don't have closed form solutions. For the same reason, finite differencing is used to approximate the gradients even though analytical derivatives could easily be derived.

Decoupling Techniques

Decoupling with the First-Order Reliability Method

Consider the optimization formulation for the first-order reliability analysis (Eq.

6) re-written for a coupled system (Eq. 9):

$$\text{Minimize } \beta = \sqrt{(X^*)^T (X^*)} \quad (9)$$

Subject to

$$g(X', u(X')) = 0$$

(Here state variables, $u(X')$, are added input to the limit state function as would be the case for a multidisciplinary system.)

In the context of multidisciplinary analysis, special MDO techniques may be employed. For example, Du and Chen (2002) present the basic IDF MDO formulation below:

$$\text{Minimize } \beta = \sqrt{(X^*)^T (X^*)} \quad (10)$$

Subject to

$$g(X', u) = 0$$

$$u = u(X')$$

With this formulation, though the end product is probabilistic information ($P_f = \Phi(-\beta)$), the most probable point (MPP), X' , is a deterministic value; therefore, the state variables at the MPP, $u(X')$, are deterministic. Thus the questions raised earlier

in the chapter, concerning how to characterize a probability density function of u , do not apply.

Both the coupled formulation (Eq. 9) and the uncoupled formulation (Eq. 10) were solved using a standard sequential quadratic programming (SQP) algorithm. The solutions are compared with one another as well as with Monte Carlo and FORM using the Rackwitz-Fiessler (R-F) method on the coupled system. The results are given in Table 1.

Table 1: Comparison of Coupled vs. Uncoupled FORM Methods

Method	β	P_f	Number of disciplinary analyses [*]
Monte Carlo	$\Phi^{-1}(.0557) = 1.5919$.0557	180,186
FORM with R-F Method	1.6252	.0521	324
FORM with SQP Coupled System (Eq. 9)	1.6252	.0521	1840
FORM with SQP Uncoupled System (Eq. 10)	1.6252	.0521	370

*The number of analyses includes finite difference runs to approximate the gradient.

From Table 1, it may be seen that all FORM algorithms produced the same result, regardless of whether it was applied to the coupled system (traditional “black box”

approach) or the uncoupled system (IDF approach). However, using the uncoupled IDF formulation netted a five-fold reduction in the total number of function evaluations over that of the coupled system. (Similar results were found by Du and Chen, 2002.) The computational effort is similar to that of the RF-FORM method, which typically converges more quickly than sequential quadratic programming. Further research may explore the possibility of using a decoupling technique in conjunction with the RF-FORM; at this point, it is unclear how the compatibility requirement (i.e. $u = u(X')$) should be enforced within the Rackwitz-Fiessler Newton step algorithm.

Decoupling with First Order Second Moment Methods

One method of characterizing the auxiliary variables is by applying a first-order, second moment (FOSM) method (Haldar and Mahadevan, 2000) on the coupled system. Mean values and standard deviations may be estimated for a function, $f(x)$, from equations (11) and (12) below.

$$\mu_f \approx f(\mu_x) \quad (11)$$

$$\sigma_f^2 \approx \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 \text{Var}(x_i) \quad (12)$$

If a finite difference process is used, $n+1$ (where n is the number of variables in x) evaluations of the function, f , are needed. This leads to $(n+1)*(Y)$ disciplinary function calls, where Y is the number of disciplinary analyses required in the fixed-point iterative process for system convergence.

FOSM may be applied directly to system output variables (e.g. limit state functions) to determine the probability of failure. For a limit state, g , the probability of failure, P_f , may be estimated in accordance with Eq. 13.

$$P_f = P(g < 0) = \Phi\left(\frac{-\mu_g}{\sigma_g}\right), \quad (13)$$

where μ_g and σ_g are found from equations (11) and (12) above. Although this method is computationally inexpensive, it is also not often very accurate, especially for non-linear limit states when the failure region is far from the mean.

A partial FOSM method may be combined with more accurate probabilistic analysis techniques through a decoupled approach. The same computational effort used to derive μ_g and σ_g for the system output, g , could be used to derive μ_u and σ_u for the system state variables, u . If a distribution (e.g. normal) is then assumed, the state variables become ordinary random variables, enabling traditional probabilistic analysis methods (e.g. sampling methods, first-order reliability methods, etc.) to be applied at the discipline level. This avoids further convergence loops for system compatibility.

Both the direct FOSM and the partial FOSM approach rely on significant assumptions, namely that the first order approximation for the mean and standard deviation is adequate and that the variables follow a normal distribution. In general, FOSM ignores any affect of non-linearity in the response function and any non-normal behavior of input variables. However, with the partial FOSM approach, these assumptions only apply to the state variables. Known distributions for other input parameters and the effect of non-linearity in the response function for the output variable may still be captured by more sophisticated probabilistic analysis tools.

Two partial FOSM techniques are employed on the example system. Both techniques use FOSM on the coupled system to estimate the mean and standard deviations of the state variables; they assume the state variables follow a normal distribution. The first method adds a Monte Carlo analysis of discipline 1 to determine the probability of failure, P_f . The second method adds a RF-FORM analysis of discipline 1 to determine P_f . These methods are compared with Monte Carlo, RF-FORM analyses, and FOSM analyses on the coupled system. Results are given in Table 2.

Table 2: Comparison of Coupled vs. Uncoupled FOSM Methods

Method	β	P_f	Number of disciplinary analyses*
Monte Carlo	$\Phi^{-1}(.0557) = 1.5919$.0557	180,186
Rackwitz-Fiessler (R-F) FORM	1.6252	.0521	324
FOSM	$\frac{\mu_{g_1}}{\sigma_{g_a}} = \frac{.500}{.3001} = 1.6661$.0478	126
FOSM with Monte Carlo	$\Phi^{-1}(.0550) = 1.5982$.0550	10,126
FOSM with RF FORM	1.6251	.0521	148

*The number of analyses includes finite difference runs to approximate the gradient.

As expected, the FOSM method on the system is not very accurate. However, using the partial FOSM method with RF-FORM increases the accuracy with very little additional computational effort. When RF-FORM is applied to the system, 324 function

calls are needed. This is due to the fixed-point convergence looping in both the evaluation of the limit state and for the gradient. For the partial FOSM/R-F method, no system convergence loops are needed, resulting in a significant computational savings. The same is true for Monte Carlo analysis. Although ten thousand iterations are called for, each system analysis call requires approximately 18 function calls in the convergence process. This is not needed when parameters of the state variable, $u_{2,1}$, are approximated with FOSM; in this case, each Monte Carlo run calls for a single disciplinary analysis. In this example, using the partial FOSM results in significant computational savings without adding detectable inaccuracy. This may not be true for all systems.

Decoupling with Gibbs Sampling

The final strategy is based on the same principles of convergence used in Gibbs sampling, a Markov Chain Monte Carlo methodology used frequently in Bayesian inference applications (Gilks et al, 1996). In Gibbs sampling, sample points are chosen from conditional probability distributions. This concept may be used to obtain samples for the state variables of a multidisciplinary analysis. In this case, the conditional probability is given by the applying the appropriate disciplinary analysis. This becomes an academic example of Gibbs sampling since the conditional probability ‘distribution’ is a discrete number.

For example, consider the two-discipline mathematical multidisciplinary analyses. The probability density functions of the system input variables (x_1 , x_2 , and x_s) are known. These variables are independent, so traditional Monte Carlo sampling may be used to generate their samples. The state variables ($u_{1,2}$ and $u_{2,1}$) are dependent on the input

variables and one another. This dependency is given by the appropriate disciplinary analysis; for a given sample of x_1 , x_3 , and $u_{2,1}$, the conditional probability of $u_{1,2}$ is a deterministic value determined by analysis 1. A psuedo-code for the example problem is given below:

Initiate : $x^0 = \{x_1^0, x_2^0, x_3^0, x_4^0, x_5^0\} = \mu_x$

$u^0 = \{u_{1,2}^0, u_{2,1}^0\}$ (These could be selected by running a full system

analyses on x^0 , but this is not required.)

Repeat {

Obtain x^i by sampling from $f_x(x)$ (Traditional Monte Carlo sampling)

Obtain $u_{1,2}$ by applying analysis 1 on x^i and $u_{2,1}^{i-1}$

Obtain $u_{2,1}$ by applying analysis 2 on x^i and $u_{1,2}^i$

$i = i + 1$

}

Here each Monte Carlo ‘run’ requires only two function evaluations, one for each disciplinary analysis. The Gibbs method does not enforce multidisciplinary system convergence at each run; instead, it takes advantage of decoupling, relying on the sampling process to produce compatibility in the limit. Fig. 4 depicts the relative convergence of the Gibbs sampling methodology with traditional Monte Carlo on the coupled system. At first glance, the convergence properties of the Gibbs method and traditional Monte Carlo appear similar. However, because each Gibbs run requires much

less computational effort (2 function evaluations versus 18 for traditional Monte Carlo), the Gibbs method is more efficient and leads to the same result.

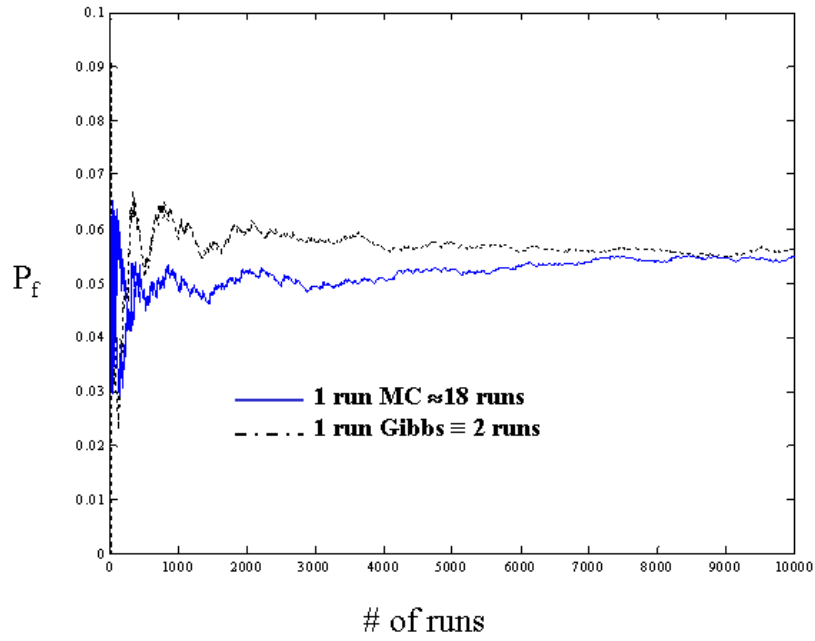


Figure 4: Convergence Comparison of Gibbs vs. Basic Monte Carlo

Summary

In this chapter, strategies for taking advantage of decoupling in performing probabilistic analysis of multidisciplinary systems have been evaluated. The first strategy is to solve the first-order reliability problem according to current MDO formulations. The second strategy is based on using first-order second moment approximations for the parameters of state variables in conjunction with traditional probabilistic methods such as Rackwitz-Fiessler FORM or Monte Carlo. The final strategy uses Markov Chain Monte Carlo sampling.

Each of these ideas has promise but needs to be examined in more detail. For example, there are a number of other MDO methods that could be applied to the first-order reliability formulation. In addition, gradient approximations are a key factor when using either FORM or FOSM. Decoupling should be able to be exploited to a greater extent in this area. Another topic for future research could examine ways to combine the Gibbs sampling approach with efficient Monte Carlo schemes such as importance sampling (Haldar and Mahadeven, 2000). Finally, the techniques should be evaluated on a number of different multidisciplinary problems to evaluate their robustness.

CHAPTER V

COMPLEX MULTIDISCIPLINARY SYSTEMS

In the previous chapter, decoupling strategies for probabilistic analysis were investigated for integrated systems. These strategies were analyzed at a conceptual level for a simple mathematical example system. In real systems, the complexity of interdisciplinary coupling and the computational effort required for the individual disciplinary analyses raises the bar for probabilistic analysis and optimization.

In this chapter, probabilistic analysis is undertaken for a Navy hypersonic missile system involving five disciplines. An intermediate build of the Integrated Hypersonic Aeromechanics Tool (IHAT) (Baker et al, 2002) is used for the multidisciplinary analysis.

From this application, it becomes evident that new strategies are needed before probabilistic analysis and optimization tools can effectively be combined in the analysis. This is even truer for the newest IHAT model, which incorporates fidelity upgrades in certain disciplines. Thus, in addition to the probabilistic analysis demonstration, this chapter highlights directions for future research in developing computationally efficient methods for probabilistic optimization of complex multidisciplinary systems.

Integrated Hypersonic Aeromechanics Tool (IHAT)

The Office of Naval Research is currently pursuing a multi-disciplinary analysis capability for the design of hypersonic weapons systems, leading to the development of

an Integrated Hypersonic Aeromechanics Tool (IHAT) (Baker et al, 2002). This ‘tool’ is an analysis model, which integrates standalone disciplinary components (consisting of either commercial software or in-house legacy codes) through their data streams. Phoenix integration’s *Model Center™* software (Phoenix Integration, 2002) was used as the integration core which links the disciplinary analyses. An experimental vehicle built in the 1970’s, the air-launched low volume ramjet (ALVRJ), is being used as a test case (Tetens, 1977).

Intermediate Build 0.5

The intermediate version (build 0.5) of the IHAT model integrates 5 disciplinary components: Geometry (using commercial software PATRAN), Aerodynamics (using Air Force legacy code S/HABP Mark V), Propulsion (using legacy code PERFORM), and Mass (an interim code which computes fuel burn along a simulated trajectory.) The coupling relationships between the analyses are depicted in Fig. 1. The geometry component scales the ramjet based on five geometric parameters: length, diameter, inlet area ratio, nozzle area ratio, and fin height. These five parameters are essentially the system design variables. The output of this module is a geometry file needed for the aerodynamics analysis and several calculated areas necessary for the propulsion analysis (combustor area, nozzle area, etc.) The Aerodynamics module analyzes the ALVRJ geometry at different mach levels, angles of attack, and control surface deflections. The output for this module is a database that includes coefficients for axial force, normal force, and pitching moment at the various flight conditions. Similarly, the Propulsion module creates a database at various conditions (mach level, altitude, angle of attack, and

engine efficiency ratio.) This database includes calculated values for thrust coefficient, fuel flow rate, and thrust. For the current model, the structural analysis is a simple parametric equation used to relate geometric effects on the structural weight. The output from this analysis is structural and empty weight and center of gravity. The Mass module computes fuel burn along a simulated trajectory. Inputs for this analysis include the databases from the propulsion and aerodynamics analyses, vehicle mass information, and a trajectory file. Outputs include maximum control surface deflections, stability margin, and remaining fuel. Finally, the Results component gathers the data to calculate fuel and volume margins.

Improved Model: Build 1

This version of the IHAT analysis enhances build 0.5 with added analyses and higher fidelity modules. A structural module that employs MSC NASTRAN to perform stress analysis and structural sizing replaces the parametric model of Build 0.5. The aerodynamics module is replaced by a variable fidelity analysis, which uses S/HABP (Gentry et al, 1973) for low fidelity aerodynamic analysis and OVERFLOW (Buning et al, 1998) for high fidelity analysis. A thermal module is added based on NASA's MINIVER program. (Build 1 brings approximately a twenty-fold increase in the computational effort over Build 0.5, mostly due to the NASTRAN structural sizing analysis.)

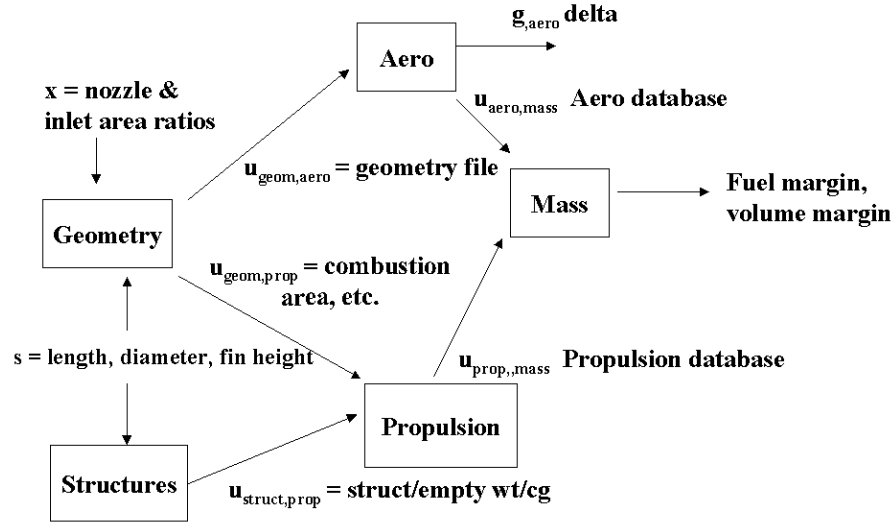


Figure 1: IHAT Inter-disciplinary Information Flow

Probabilistic Analysis on the IHAT Model

Probabilistic analysis was performed on the IHAT build 0.5. In this analysis, five random input variables are considered as shown in the Table 1. Note that all five random variables relate to a single discipline: geometry. In this problem, all five are also design variables. However, it is easy to include random variables from other disciplines in the analysis. The mean values are based on the ALVRJ test case. Normal distributions with a 10% coefficient of variation were chosen simply for the sake of demonstration.

Table 1: Random Variables

Variable	distribution	mean	cov
length	Normal	190	0.1
radius	Normal	20	0.1
ar-inlet	Normal	0.55	0.1
ar-nozzle	Normal	0.7	0.1
fin height	Normal	19.9	0.1

The goal of the analysis was to find the probability that the fuel margin exceeded 100 lb. The fuel margin is the output of the five-discipline analysis shown in Fig. 1. The first order reliability method was used. As a reminder, the Rackwitz-Fiessler algorithm is given again in Eq. 1:

$$X_{k+1}^* = \frac{1}{|\nabla g(X_k^*)|^2} [\nabla g(X_k^*)' (X_k^*) - g(X_k^*)] \nabla g(X_k^*) \quad (1)$$

The gradient vector, $\nabla g(X_k^*)$ was estimated from a simple forward difference procedure. Six iterations were required to converge to the MPP with β equal to 1.64. This corresponds to a probability of $\Phi(-1.64) = .05$. It should be noted that finding the gradient required $N+1$ (N is the number of random variables) ‘runs’ of the multi-disciplinary analysis. Thus, the total calculation required $6 * (5+1) = 36$ evaluations of the multi-disciplinary analysis. This is still much better than a Monte Carlo analysis, which would require 1900 simulations to establish a 5% probability of failure within a 20% error. Although achievable for Build 0.5, this number of iterations will likely not be

acceptable for the more expensive analyses involved in Build 1. In addition, adding an optimization outer loop would appear to make the problem even less tractable.

Direction for Future Work

Future research should be directed toward synthesizing probabilistic analysis techniques with the IHAT multi-disciplinary optimization (Build 1), enabling design decisions based on performance reliability and cost. This work should consider the effects of various sources of uncertainty on system performance reliability, as discussed below.

Uncertainty in System Variables

A straightforward probabilistic analysis for the IHAT model considers the uncertainty in system variables. In other words, system parameters are not known deterministically, but are assumed to have some probability density function describing their variability. This type of uncertainty is inherent and for the most part irreducible. An example formulation that accounts for this type of uncertainty follows:

$$\text{Maximize Expected (mean) Fuel Margin} = f(\mu_{\text{design}}) \quad (8)$$

where μ_{design} is the mean for the design variables.

$$\text{Subject to } P(g_1 = \text{Volume Margin} < 0) < P_{f1} \quad (8a)$$

$$P(g_2 = 30 - \text{Max delta} < 0) < P_{f2}$$

$$P(g_3 = 30 + \text{Min delta} < 0) < P_{f3}$$

The fuel margin here represents available weight for payload. The volume margin is the additional volume available in the fuel tank, and the min and max delta are extreme

control surface deflections. Alternately, the constraints may be considered as a single system failure constraint:

$$P\{(g_1 < 0) \cup P(g_2 < 0) \cup P(g_3 < 0)\} < P_{fs} \quad (8b)$$

where g_1 , g_2 , g_3 are the same functions defined above, \cup represents the union of constraint failure events, and P_{fs} is the acceptable probability of system failure. The significance of the union is that it defines system failure as the case when any one constraint has not been satisfied. This is similar to the multiple failure mode approach used in Chapter III.

Although the formulation is fairly straightforward, the multidisciplinary nature of both the objective and constraint functions complicates the solution of this optimization problem. The computational time of a single system analysis is multiplied, first by the number of iterations required for probabilistic analysis for each constraint, and second by the number of optimization iterations. The computational expense issue will be compounded if iterative feedback loops become a part of the system analysis with future models. Furthermore, the problem is exacerbated by the need for gradient calculations for both optimization and probabilistic analysis. However, there are many developments in the field of multidisciplinary analysis and multidisciplinary optimization that may have application to probabilistic MDO. These include approximation modeling, efficient derivative approximations, distributed problem formulations, and multi-level optimization algorithms. Future research should explore an array of these methods to facilitate the solution of the above probabilistic MDO formulation.

Statistical Uncertainty

Another type of uncertainty known as statistical uncertainty arises from sparse data on the system variables. Rarely are system variables so well understood that engineers are precisely able to characterize them with probability distribution functions. Instead they typically use whatever data is available to make predictions of the distribution parameters (e.g. mean and standard deviations) for these variables. These predictions are made with a degree of uncertainty, which can be quantified if the parameters themselves are treated as random variables. For example, if missile length is a system variable with inherent variability, it may be represented by a distribution with a specific mean, μ_{length} and standard deviation, σ_{length} . To the extent that μ_{length} and σ_{length} are uncertain, they may have their own random distribution parameters, μ_{μ} , σ_{μ} , μ_{σ} , and σ_{σ} . In this case, the system failure probability will also be a random variable, and confidence bounds may be used to articulate the uncertainty in this calculation. Note that statistical uncertainty arises only from a lack of data regarding the system variables; it may be reduced through additional testing so that randomness in system variables is better understood.

Model Uncertainty

Another issue for system reliability assessment is model uncertainty. Design of complex systems is typically progressive in nature, stepping up from lower-fidelity, less expensive analysis to higher fidelity, more expensive analysis. Thus, during the first stages of conceptual design, the low-fidelity analysis tools typically have a large degree of model uncertainty, which is independent of parametric uncertainty. This type of

uncertainty, if properly quantified, could ideally play an important role in model selection and refinement during the various stages of design. When the model uncertainty for a particular low-fidelity disciplinary analysis has a minor effect on the uncertainty of system performance, there is little cause to expend additional resources to upgrade to a higher fidelity analysis. Conversely, if a system is very sensitive to the uncertainty of a given disciplinary model, increases in its fidelity will significantly reduce the uncertainty of system performance. Future research should explore methods for evaluating model uncertainty as part of a strategy for choosing disciplinary model fidelity based on system requirements.

CHAPTER VI

CONCLUSION

Summary

The future of aerospace design is dependant on the ability to analyze and improve complex multidisciplinary systems. In this environment, a methodology to evaluate cost and risk, and to facilitate design trade-offs, is of utmost value. Three key principles that apply in this effort include probabilistic analysis, optimization, and multidisciplinary integration. Developing tools that combine these three principles is a daunting but necessary task. In this vein, some practical techniques for probabilistic analysis and optimization of multidisciplinary systems have been investigated in this study.

First, probabilistic optimization was performed for the global, geometry design of a reusable launch vehicle. This application used a response surface methodology to integrate a three-discipline system including geometry, weights, and aerodynamic analyses. Reliability was defined by the probability of meeting performance constraints and evaluated by the First-order Reliability Method (FORM). Cost and risk trade-offs were accomplished through a probabilistic optimization formulation. The technique proved to be computationally efficient, yield believable results, and bring added value to deterministic optimization. The limitation, however, is that the approach relied on a very low-fidelity system analysis (i.e. the response surfaces). While possible adequate for the earliest stages of conceptual design, these methods need to be elevated for higher fidelity multidisciplinary models.

Next, probabilistic optimization was performed for the structural sizing of a liquid hydrogen tank. Here probabilistic data flowed down from the global design to the local level. In addition, the optimization was constrained by the system reliability, which was evaluated for multiple failure modes. In stepping from the global design of chapter two to the local design of chapter three, this application demonstrated a systematic process for linking design at differing levels.

In Chapter IV, three concepts were investigated to facilitate probabilistic analysis on a multidisciplinary system with feedback using a decoupling strategy. These include: (1) using MDO methods with the first-order reliability formulation, (2) using first-order second moment methods to characterize state variables, and (3) using Gibbs sampling to enforce system compatibility in the limit rather than for each iteration. Finally, in Chapter V, probabilistic analysis was undertaken for an intermediate build of a high fidelity analysis of a Navy missile system. This application highlighted many of the computational challenges in implementing probabilistic optimization in complex multidisciplinary systems.

The goal of this study was to present a strategy for conceptual system design that simultaneously addresses both cost and risk requirements. The general methodology offered is probabilistic, multidisciplinary optimization. The methodology was successfully demonstrated for low-fidelity analyses (possibly sufficient for conceptual design), but much work remains before full probabilistic optimization can be employed with high fidelity systems.

Future Needs

Aerospace design relies on multidisciplinary models. Efficient methods are needed to incorporate probabilistic optimization for these models without sacrificing the fidelity appropriate for advanced stages of design.

In Chapter IV, a decoupling strategy was investigated for multidisciplinary systems with feedback coupling. Although this strategy shows promise, additional research is recommended to refine and test the techniques. For example, the Rackwitz-Fiessler iterative algorithm is typically faster than other algorithms used to solve the first-order reliability formulation; research should be directed toward combining this method with decoupling MDO strategies. In addition, MDO formulations other than IDF (i.e. CO, BLISS) could be investigated for their benefit in solving the first-order reliability problem. As another example, the Gibbs sampling approach resulted in a nine-fold improvement over traditional Monte Carlo for the demonstration problem. Perhaps the Gibbs sampling approach can be combined with efficient Monte Carlo techniques (e.g. adaptive importance sampling, etc.) for further improvement. Another tactic that should be considered is using alternate gradient approximation methods. As shown in both the mathematical system of Chapter IV and the missile application in Chapter V, gradient computations have a significant impact on the total computational effort.

Chapter V offered a specific application (missile system design) for future research involving a complex, high-fidelity multidisciplinary system model. The need for optimization tools that will account for various types of uncertainty (uncertainty in system variables, statistical uncertainty, and model uncertainty) for these types of systems is self-evident. However, serious challenges remain with respect to the

computational effort and robustness of traditional “black box” approaches to this problem. Within the multidisciplinary optimization (MDO) field, several techniques are available which may be effective in streamlining probabilistic MDO. This is another recommended area for future work.

Finally, assessing the impact of reducible error sources (such as statistical uncertainty, which can be reduced through testing, and model error, which can be reduced by using higher fidelity models) would be a valuable tool enabling engineers to make intelligent choices about how to invest resources. It is recommended that future research be directed toward these kinds of tools as well.

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